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Committee Search Design

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Committee Search Design^{*}

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Abstract

This paper studies the design of committee search procedures. In each time period, a set of candidates of fixed size arrives, and committee members vote whether to accept a candidate out of this set or to continue costly search. We examine the implications of different sample sizes per period on acceptance standards and welfare, and we derive the welfare-maximizing number of candidates per period for small magnitudes of search costs. There is a trade-off between the expected value of a candidate conditional on stopping and the expected search costs. The resolution of this trade-off depends on the voting rule and the shape of the search cost function. In particular, we show that, for all cost functions and all qualified majority voting rules other than unanimity, welfare is increasing in the number of candidates per period if the magnitude of search costs is sufficiently small. This result stands in contrast to the classic finding for the single decision-maker case where the evaluation of multiple candidates per period does not improve welfare relative to reviewing one candidate at a time if there are no economies of scale in the simultaneous evaluation of multiple candidates. *Keywords:* Committee Search; Sequential Search; Multiple Options

JEL Classification: D71, D83

1 Introduction

Academic hiring is mostly conducted by search committees. Often several candidates are reviewed simultaneously after the application deadline has been reached, and the committee either selects one suitable candidate or the hiring process starts over if neither of the candidates satisfied the committee’s acceptance standards.¹ So far, the literature on committee search has mainly focused on a search process where candidates are reviewed “one at a time”, meaning, hiring is conducted on a rolling basis. Define this search procedure as *single-option sequential search*.² In this paper, we consider search technologies where committees evaluate a fixed number of candidates simultaneously in each time period, which we denote by *multi-option sequential search*. The number of candidates who are simultaneously evaluated per period will be denoted by the *sample size*.

We compare single-option sequential search and multi-option sequential search with different sample sizes in terms of acceptance standards and the ex ante utilitarian welfare for the search committee. Moreover, we derive the welfare-maximizing sample size per period if search costs are sufficiently small.

Under multi-option sequential search, committee members can directly compare candidates. This has two implications: On the one hand, the expected value of a candidate conditional on hiring increases with the sample size. On the other hand, the probability of hiring a particular candidate decreases with the sample size, and, thus, the expected search costs are altered. Generally, there is a trade-off between these two objects that determine the committee’s welfare. The resolution of this trade-off depends on the voting rule and the specification of search costs associated with the simultaneous evaluation of multiple candidates. We find that under unanimity voting, the ranking of the discussed search procedures depends on how the search costs vary with the sample size. In contrast, under qualified majority voting distinct from unanimity, this sensitivity to the shape of the cost function partly disappears. In this case, we show that, independently of the shape of the cost function, reviewing more candidates per period of time improves wel-

¹See for example the descriptions of the hiring processes of the [Columbia University in the City of New York \(2016\)](#), [The University of Arizona \(2019\)](#) or [The University of California, Berkeley \(2019\)](#).

²In the search literature, this is mostly denoted as *sequential search*.

fare as long as the magnitude of the search costs is sufficiently small.

Note that search conducted by a single decision-maker is a special case of a committee operating under unanimity voting. Our results imply that the classic finding for the single decision-maker case where evaluating multiple candidates at a time instead of only one does not improve welfare if there are no related economies of scale (see e.g. [Manning and Morgan \(1985\)](#)) does not extend to a committee operating under a qualified majority voting other than unanimity. In other words, unless the search committee operates under unanimity voting, the economic trade-offs determining a good search rule for committees are significantly different from those in the single decision-maker case. Thus, treating the committee as a single agent would lead to systematically wrong predictions.

In our model, a search committee consisting of at least one member seeks to hire one candidate. The committee reviews in each time period a fixed number of candidates $K \geq 1$ simultaneously. The time horizon is infinite, and rejected candidates cannot be recalled.³ The committee members' preferences feature independent private values. For every member, the value of a candidate is a random variable, which is distributed independently and identically across time, members, and candidates. Each committee member observes his or her own value realization for every candidate and has distributional knowledge about the other members' values.

We consider a class of voting rules where each member may either vote for one of the available candidates or may opt to continue search. A candidate is then hired if and only if the number of votes he or she receives exceeds a qualified majority threshold ranging from simple majority to unanimity. This class of voting rules is frequently used in practice, making it a natural choice when adopting an approach that is positive with regard to the voting rule, but normative with respect to the search technology.⁴ For example, when abstracting from abstention, the

³Note that multi-option sequential search corresponding to the case in which $K > 1$ can also be interpreted as delayed voting: Suppose that one candidate per period arrives. Then, simultaneously evaluating K candidates in some round of the dynamic search procedure can be viewed as taking voting decisions only every K periods instead of every single period. In other words, choosing the sample size K can be viewed as selecting voting times. We thank Olivier Compte for suggesting this interpretation.

⁴Since members have to decide about more than two alternatives under multi-option sequential search, other voting rules are also conceivable. We discuss this point in [Section 7](#).

default voting rule for collective decisions by the general assembly of registered associations in Germany prescribes that any resolution requires the support of a simple majority, independently of the number of alternatives (cf. [Bundesrepublik Deutschland \(2019\)](#)).

If a candidate is hired, search stops; otherwise, search continues, and each committee member bears an additive search cost $c \cdot h(K) > 0$. We restrict the committee members' voting strategies to symmetric and neutral⁵ stationary Markov strategies. Then, a member votes in favor of some candidate if and only if the candidate's value is the highest among all observed K values and it exceeds some cutoff representing the member's acceptance standard. Acceptance standards coincide with welfare because values are private.⁶

To begin with, we prove the existence and uniqueness of a symmetric and neutral stationary Markov equilibrium for all $K \geq 1$ and for all qualified majority voting rules including unanimity voting. The uniqueness of equilibrium is shown for value distributions that admit a log-concave density. In the subsequent comparison of search procedures, we maintain this distributional assumption.

Consider two search procedures with $K' \geq 1$ and $K \geq 1$ candidates per period, and suppose that $K' > K$. First, we study the case of unanimity voting in detail. We find that if the cost function h satisfies $\frac{h(K')}{K'} \geq \frac{h(K)}{K}$, i.e., the search costs per candidate are weakly higher if there are K' versus K candidates at a time, evaluating K candidates per period yields higher acceptance standards and welfare than reviewing K' candidates at a time.⁷ Intuitively, given some acceptance standard, the expected value of a candidate conditional on stopping is higher if there are K' than if there are K candidates per period. However, at the same time, expected search costs are also higher because the probability of hiring a particular candidate is lower and the function h satisfies $\frac{h(K')}{K'} \geq \frac{h(K)}{K}$. We show that the increase in the expected value conditional on stopping is limited and that the overall trade-off is resolved in favor of the search procedure with K candidates per period. This result implies in particular that single-option sequential search is welfare-maximizing if the function h meets the condition $\frac{h(K')}{K'} \geq h(1)$ for all

⁵A strategy is *neutral* if it does not condition on the identity of the candidate.

⁶To be precise, this holds if and only if the equilibrium cutoff is interior.

⁷This result does not require the density of the value distribution to be log-concave.

$K' > 1$. In contrast, if $\frac{h(K')}{K'} < \frac{h(K)}{K}$, reviewing K' candidates at a time yields higher welfare than evaluating K candidates in every period if the magnitude of search costs quantified by the parameter c is sufficiently small. Here, if c is small, acceptance standards are close to the upper bound of the support of the value distribution. Hence, while the probability of hiring a particular candidate is higher under the search technology featuring K candidates per period, it is low for both search procedure. Therefore, if c is sufficiently small, expected search costs are actually lower if there are K' candidates at a time because h is assumed to satisfy $\frac{h(K')}{K'} < \frac{h(K)}{K}$. In addition, as before, the expected value conditional on stopping for a sample size K' is not lower than the respective value for a sample size K . Moreover, if the search costs per candidate are minimal for some form of multi-option sequential search distinct from single-option sequential search and for exogenous reasons at most $\bar{K} < \infty$ candidates can be reviewed simultaneously, the two discussed results imply together that multi-option sequential search with a sample size that coincides with the smallest minimizer of the search costs per candidate is welfare-maximizing as long as the magnitude of the search costs c is sufficiently small. Extensions to interdependent values and correlated values contained in Section 7 show the robustness of these findings.

Second, we investigate qualified majority voting rules that do not require full unanimity. Again, consider two search procedures with $K' \geq 1$ and $K \geq 1$ candidates per period, and assume that $K' > K$. We find that reviewing K' candidates at a time yields a higher welfare than evaluating K candidates per period for all cost functions h as long as c is sufficiently small. Thus, the sensitivity to the shape of the cost function h that we find for the unanimity rule partly disappears. To prove this result, we first establish that the ranking of the expected values conditional on stopping from the unanimity voting case carries over to qualified majority, meaning, the respective expected value is higher if there are K' compared to K candidates per period. Then, we show that if c is sufficiently small, this increase in the expected value conditional on stopping outweighs the potential rise in expected costs.⁸ Furthermore, this result has the following implication for the

⁸Depending on the shape of the cost function h , expected search costs might also decrease. Of course, this only reinforces our reasoning.

welfare-maximizing sample size per period. Suppose that for exogenous reasons at most $\bar{K} < \infty$ candidates can be evaluated simultaneously in each period of time. Then, whatever the shape of the cost function h , multi-option sequential search with $\bar{K}$ candidates at a time is welfare-maximizing as long as the magnitude of the search costs c is sufficiently small.

Consequently, the comparison of single-option to various forms of multi-option sequential search differs considerably if the search committee operates under qualified majority voting instead of unanimity voting. This is the main qualitative insight of our paper. Moreover, as alluded to above, our results imply in particular that the conclusions for search conducted by a single decision-maker which is a special form of committee search with unanimity voting do not carry over to committee search with qualified majority voting.

Our theoretical results are empirically relevant for hiring problems. [Abowd and Kramarz \(2003\)](#) as well as [Kramarz and Michaud \(2010\)](#) empirically study the magnitude and the functional form of the hiring costs in France.⁹ Since their definition of hiring costs excludes training costs, we believe that these empirical findings generate plausible proxies for the search costs in our model. For long-term contracts and highly skilled jobs, they find that the magnitude of costs is rather small and that the cost function is increasing and concave.¹⁰ Therefore, we conclude that our theoretical findings, which mostly focus on the case of small magnitudes of costs, apply to hiring problems in practice. Moreover, the empirical finding that the cost function is increasing, but concave suggests that multi-option sequential search improves welfare relative to single-option sequential search for the unanimity as well as for all qualified majority voting rules. This discussion demonstrates that our theoretical findings have empirically relevant implications for hiring problems.

The paper is organized as follows: Section [2](#) reviews the related literature, section [3](#) introduces the model, and section [4](#) proves the existence and uniqueness of the equilibrium. Section [5](#) treats the unanimity voting case, and section [6](#) contains the results for qualified majority voting rules. The next section [7](#) contains the exten-

⁹We thank Simon Gleyze for making us aware of these papers.

¹⁰Notice that these estimates are particularly relevant because hiring by committee is mostly done for long-term contracts and highly skilled jobs.

sions to interdependent and correlated values, and we discuss other voting rules. The final section 8 concludes. Appendix A contains the proofs, and Appendix B derives expressions for the probability of approving a particular candidate and the expected value conditional on stopping.

2 Related Literature

Our paper contributes to the growing literature on committee search where a committee conducts search dynamically over time.¹¹ Albrecht et al. (2010), Compte and Jehiel (2010), and Moldovanu and Shi (2013) study different aspects of committee search while focusing exclusively on single-option sequential search. For example, Albrecht et al. (2010) study the implications of different voting rules or committee sizes while holding the search technology, i.e., single-option sequential search, fixed. In contrast, our paper focuses on the effect of different search procedures on acceptance standards and welfare given the voting rule. Therefore, we contribute to the literature on committee search by introducing multi-option sequential search and comparing these alternative search protocols to single-option sequential search in terms of acceptance standards and welfare.

We know of only one other contribution that is concerned with the comparison of different search technologies in the committee search environment.¹² In independent work, Cao and Zhu (2019) compare single-option sequential search with simple majority voting to a fixed-sample-size search technology that can be described as follows: First, the committee determines the total sample size via the random proposer mechanism. Then, in each period, one alternative is drawn until the predetermined sample size is reached. Finally, the committee selects an alternative according to plurality voting. Cao and Zhu (2019)’s main insight is that the finding from the single decision-maker setting that single-option sequential search always dominates fixed-sample-size search, as for example argued in Rothschild

¹¹The static case of committee decision-making has also been analyzed in depth, cf. the survey by Li and Suen (2009).

¹²In the literature on auctions, the comparison between different selling technologies has been studied before. Wang (1993) compares auctions to posted-price selling in terms of revenue and prices and finds that the ranking of the two technologies depends on the seller’s auctioning costs and on the steepness of the marginal revenue curve.

(1974), does not extend to the committee search setting. While [Cao and Zhu \(2019\)](#) independently ask a similar research question to ours, they study a conceptually different search technology, inducing different results driven by different effects. Therefore, we view our paper to be complementary to their work.

In the literature on search conducted by a single decision-maker, not only single-option sequential search due to [McCall \(1970\)](#) but also other search technologies have been discussed and contrasted.¹³ In [Morgan \(1983\)](#) as well as [Manning and Morgan \(1985\)](#), search is conducted by a single decision-maker, and they consider general classes of search procedures, where, in each period, the single agent decides how many alternatives to draw in the following period if search continues and whether to stop search in the current period. Therefore, multi-option sequential search conducted by a single decision-maker is part of the search technologies studied in [Morgan \(1983\)](#) as well as [Manning and Morgan \(1985\)](#).

[Morgan \(1983\)](#) derives properties of the optimal sample size in each time period depending on the searcher’s recall, time horizon, and outside option, but he does not analytically identify conditions on the primitives of the model under which single-option sequential search is optimal. However, he mentions numerical simulations indicating in particular that single-option sequential search might not be optimal if there is no recall and there are intraperiodic economies of scale in the simultaneous evaluation of multiple alternatives. To some extent, our analytical result for committee search with unanimity voting and cost functions h satisfying $\frac{h(K')}{K'} < \frac{h(K)}{K}$ with $K' > K \geq 1$ specialized to the single-agent case addresses this point.

[Manning and Morgan \(1985\)](#) show analytically that single-option sequential search conducted by a single agent is optimal if the time horizon is infinite, there is full recall, and the single searcher bears additive search costs that are increasing and convex in the number of alternatives per period. This result resembles our finding for committee search with unanimity voting and cost functions h satisfying $\frac{h(K')}{K'} \geq h(1)$ for all $K' > 1$ when specializing it to the single-searcher case. Note that [Manning and Morgan \(1985\)](#) assume full recall, whereas we assume that rejected alternatives cannot be recalled. Yet, as long as the sample size per period

¹³See for instance [Stigler \(1961\)](#), [Rothschild \(1974\)](#), and [Burdett and Judd \(1983\)](#).

does not depend on calendar time as it is the case under single-option as well as multi-option sequential search, in the single-agent case, the no recall assumption is without loss.¹⁴ Therefore, our finding for committee search with unanimity voting and functions h satisfying $\frac{h(K')}{K'} \geq h(1)$ for all $K' > 1$ specialized to the single-agent case can be derived from [Manning and Morgan \(1985\)](#)'s result.¹⁵

3 Model

A committee consisting of members $\mathcal{N} := \{1, \dots, N\}$ with $N \geq 1$, who are indexed by i , seeks to hire one candidate. In each discrete period of time t , a set of candidates $\mathcal{K} := \{1, \dots, K\}$ with $1 \leq K < \infty$ arrives. If $K = 1$, we call the resulting search procedure single-option sequential search, whereas, if $K > 1$, the search technology is termed multi-option sequential search. K is also denoted as sample size.

Preferences feature independent private values. For each committee member $i \in \mathcal{N}$, the value of hiring candidate $k \in \mathcal{K}$ is governed by the random variable X_i^k , where X_i^k is distributed independently and identically across time periods, candidates, and members according to the cumulative distribution function F with density f . We assume that the distribution of X_i^k has full support on the bounded interval $[0, \bar{x}]$ with $\bar{x} > 0$. Let μ denote the mean of the random variable X_i^k . For all candidates $k \in \mathcal{K}$, committee member $i \in \mathcal{N}$ observes the realization of X_i^k perfectly and has only distributional knowledge about the value X_j^k that any committee member j other than i assigns to candidate k .

The timing is as follows: In every time period, member i observes a realization of the vector of random variables $(X_i^1, \dots, X_i^K)$, that is, K values. Then, members simultaneously cast a vote, voting either for one candidate k (action k) or for the option to continue search (action 0). Candidate k is hired and search is stopped if and only if the number of votes in favor of k is larger than or equal to the (qualified) majority threshold $M \in \{1, \dots, N\}$, with $M > \frac{N}{2}$.¹⁶ This class

¹⁴For single-option sequential search and a single decision-maker, this point has been made previously by [Albrecht et al. \(2010\)](#).

¹⁵However, note that our assumption on the shape of the cost function is more general.

¹⁶The assumption $M > \frac{N}{2}$ ensures that no two distinct candidates meet the (qualified) majority requirement at the same time.

of voting rules encompasses, for instance, unanimity voting corresponding to the case where $M = N$ or simple majority voting with an odd number of members, that is, $M = \frac{N+1}{2}$. If search is continued, each committee member incurs a per period cost of $c \cdot h(K) > 0$, where $h(K)$ is the value of some function $h : \mathbb{N}_+ \rightarrow \mathbb{R}_{>0}$ evaluated at K , and $c > 0$ represents a scaling parameter. Finally, we assume that the search horizon is infinite, and that rejected candidates cannot be recalled.

4 Equilibrium Analysis

Committee member i 's strategy is a sequence of functions $\sigma_i = \{\sigma_i(H_t)\}_t$, mapping from any history H_t until period t to $\Delta(\{0\} \cup \mathcal{K})$, i.e., all probability distributions over the set of actions $\{0\} \cup \mathcal{K}$ that are available in each period. As is common in the literature on committee search, we restrict strategies to be (1) Markovian, meaning, the action that member i 's strategy prescribes in period t does not depend on the entire history up to period t , but only on the evaluation of the most recent K candidates, and we focus on (2) stationary and (3) symmetric equilibria, that is, the equilibrium strategies are neither sensitive to calendar time nor to the identity of the committee member. In addition, we assume strategies to be (4) neutral, that is, they have to be invariant with respect to permutations of the candidates' labels.¹⁷ Essentially, neutrality rules out stationary and symmetric equilibria in Markov strategies in which voters coordinate on ignoring one or more candidates. Apart from conditions (1) - (4), we also impose that search terminates in finite time, excluding dominated equilibria in which all members always vote to continue search, independently of the value realizations. Subsequently, we simply write equilibrium when referring to a stationary and symmetric Markov equilibrium in neutral strategies.

Strategies that satisfy these refinements are characterized by cutoffs $z \in [0, \bar{x}]$.

More specifically, in any time period, upon observing the value realizations

$(x_i^1, \dots, x_i^K) \in [0, \bar{x}]^K$, member $i \in \mathcal{N}$ votes in favor of candidate $k \in \mathcal{K}$ if and only

¹⁷Any stationary Markov strategy can be described by a mapping $s : [0, \bar{x}]^K \rightarrow \Delta(\{0\} \cup \mathcal{K})$. A strategy s satisfies neutrality if, for all $(x^1, \dots, x^K) \in [0, \bar{x}]^K$, it holds that $s(x^{\rho(1)}, \dots, x^{\rho(K)}) = (s^0(x^1, \dots, x^K), s^{\rho(1)}(x^1, \dots, x^K), \dots, s^{\rho(K)}(x^1, \dots, x^K))$ for any permutation ρ of the set $\mathcal{K}$.

if

$$x_i^k \geq \max_{l \neq k} x_i^l \text{ and } x_i^k \geq z.$$

We call these strategies *maximum-strategies with cutoff*. In words, every member chooses the best among the K available candidates and approves this candidate whenever the associated value exceeds the cutoff, or acceptance standard, z . Intuitively, since candidates are identical ex ante and because members treat candidates in a neutral way, all candidates have the same chance to be elected from the perspective of an individual member. Consequently, no member has an incentive to vote in favor of any candidate but the best.¹⁸

Interior equilibrium cutoffs $z \in (0, \bar{x})$ solve $z = v$, where v is the continuation value implied by this strategy profile.¹⁹ The continuation value which coincides with the ex ante utilitarian welfare per committee member is given by

$$v = -\frac{c \cdot h(K)}{K \cdot \Pr(\text{candidate } k \text{ hired})} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}].$$

The continuation value amounts to the difference between the expected value conditional on stopping $\mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]$ and the expected search costs $\frac{c \cdot h(K)}{K \cdot \Pr(\text{candidate } k \text{ hired})}$. Let $Q^K(z, N, M)$ be the cumulative distribution function of the Binomial distribution with parameters N and $\Pr(X_i^k \geq z \text{ and } X_i^k \geq \max_{l \neq k} X_i^l)$ evaluated at $M-1$. Also, for any $b \in \mathbb{N}_0$ with $b \leq N$, $q^K(z, N, b)$ denotes the corresponding probability mass function evaluated at b . Further, we argue in Appendix B.2 that

$$\Pr(X_i^k \geq z \text{ and } X_i^k \geq \max_{l \neq k} X_i^l) = \frac{1}{K} [1 - F(z)^K].$$

Then, the equilibrium equation can be written as

$$z = -\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]. \quad (1)$$

¹⁸Note that mixed strategies do not arise in equilibrium.

¹⁹Boundary solutions, i.e., equilibria involving some maximum strategy with cutoff $z = 0$, may arise if the search costs $c \cdot h(K)$ are large. Subsequently, we take care of this issue.

Intuitively, acceptance standards z arising in equilibrium are calibrated in a way such that a member is indifferent between stopping and continuing search whenever the value of some candidate coincides with the cutoff z . A derivation of the equilibrium strategies and the equation characterizing the equilibrium cutoffs can be found in [Appendix A.1](#).

4.1 Equilibrium Existence

We claim that there exists an equilibrium. The reasoning in the previous part implies that there exists an equilibrium if and only if there either exists $0 \leq z < \bar{x}$ that solves equation (1), or there is a boundary equilibrium, in which the maximum-strategy with cutoff $z = 0$ forms an equilibrium.

Proposition 1. *There exists an equilibrium.*

We prove the existence of an equilibrium while making use of the intermediate value theorem. Similar existence arguments appear in [Albrecht et al. \(2010\)](#), [Compte and Jehiel \(2010\)](#), and [Moldovanu and Shi \(2013\)](#).

4.2 Equilibrium Uniqueness

We turn to the problem of equilibrium uniqueness. Apart from being of interest in itself, the uniqueness of equilibrium is important for a transparent comparison between single-option sequential search and multi-option sequential search. It turns out that the equilibrium is unique if we impose the assumption that the density f is log-concave.

Proposition 2. *If the density f is log-concave, the equilibrium is unique.*

Many well-known distributions including, for instance, the uniform distribution or the truncated normal distribution meet this requirement.²⁰

Conceptually, the proof strategy follows [Albrecht et al. \(2010\)](#), but, as discussed below, the presence of more than one candidate per period requires a substantial amount of supplementary steps that are not needed if $K = 1$. The arguments from

²⁰For a comprehensive list of distributions that admit a log-concave density, we refer to [Bagnoli and Bergstrom \(2005\)](#).

the previous parts imply that there is a unique equilibrium if and only if either equation (1) admits exactly one solution and there is no supplementary boundary equilibrium or there is a boundary equilibrium and the equilibrium equation has no solution. Rearrange equation (1):

$$\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} = \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}] - z.$$

The essential part of the proof is to establish that the left-hand side of this equation is increasing in z , whereas the right-hand side is decreasing in z . Then, the uniqueness result follows from the opposite monotonicities of the discussed functions.

First, it is straightforward to derive that the left-hand side is increasing in z . Intuitively, if the acceptance standard z increases, the probability of voting in favor of some candidate k decreases, and, hence, the probability of hiring this candidate k and the overall probability of stopping decrease as well. Thus, the expected search costs increase. Consequently, it remains to show that $\mathbb{E}[X_i^k | \text{candidate } k \text{ hired}] - z$ is decreasing in z . This claim is stated as Lemma 1.²¹ Define $S^K(z, N, M) := \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]$ to emphasize that the expected value conditional on hiring depends on K and M .

Lemma 1. *Consider any $K \geq 1$. If the density f is log-concave, the function*

$$S^K(z, N, M) - z$$

is decreasing in z .

Subsequently, we discuss the proof of Lemma 1. Introduce the following two objects:

$$\begin{aligned} \mu_a^K(z) &:= \mathbb{E}[X_i^k | X_i^k \geq z \text{ and } X_i^k \geq \max_{l \neq k} X_i^l], \text{ and} \\ \mu_r^K(z) &:= \mathbb{E}[X_i^k | X_i^k < z \text{ or } X_i^k < \max_{l \neq k} X_i^l]. \end{aligned}$$

²¹For single-option sequential search, i.e., $K = 1$, this property has been shown in Albrecht et al. (2010).

These conditional expectations capture the expected value of an arbitrary candidate $k \in \mathcal{K}$ for an arbitrary member $i \in \mathcal{N}$ conditional on approving or rejecting this candidate, respectively. We argue in Appendix B.1 that

$$\mathbb{E}[X_i^k | \text{candidate } k \text{ hired}] = w^K(z)\mu_a^K(z) + [1 - w^K(z)]\mu_r^K(z), \quad (2)$$

with $w^K(z)$ being defined as

$$w^K(z) := \sum_{l=M}^N \frac{q^K(z, N, l)}{1 - Q^K(z, N, M)} \frac{l}{N}.^{22}$$

Intuitively, conditional on stopping, the accepted candidate k might be supported or rejected by an arbitrary member. Therefore, the expected value of k conditional on stopping amounts to an average of the expected values conditional on supporting as well as rejecting candidate k . The weight $w^K(z)$ represents the expected share of members supporting k conditional on k meeting the majority requirement. Note that under unanimity voting, hired candidates must be accepted by every member. Thus, in this case, the expected value conditional on hiring simplifies to $\mathbb{E}[X_i^k | \text{candidate } k \text{ hired}] = \mu_a^K(z)$.

After some intermediate steps that are similar to those in the proof of Albrecht et al. (2010) we obtain that, for $z \in (0, \bar{x})$,

$$\frac{d\mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]}{dz} < w^K(z) \frac{d\mu_a^K(z)}{dz} + [1 - w^K(z)] \frac{d\mu_r^K(z)}{dz}.$$

Hence, the key proof step is to show that $\frac{d\mu_a^K(z)}{dz} \leq 1$ and $\frac{d\mu_r^K(z)}{dz} \leq 1$. Notice that if $K = 1$, these conditional expected values are truncated means:

$$\mu_a^1(z) = \mathbb{E}[X_i^k | X_i^k \geq z], \text{ and } \mu_r^1(z) = \mathbb{E}[X_i^k | X_i^k < z].$$

It is well-known that log-concavity of f implies the desired Lipschitz conditions on the truncated means, i.e., $\frac{d\mu_a^1(z)}{dz} \leq 1$ and $\frac{d\mu_r^1(z)}{dz} \leq 1$ (see e.g. Bagnoli and Bergstrom (2005)). However, for $K > 1$, the discussed implications are not standard because

²²This kind of representation of the expected value conditional on stopping is due to Albrecht et al. (2010).

the involved expected values conditional on rejecting or supporting a candidate do no longer constitute truncated means. To obtain that $\frac{d\mu_a^K(z)}{dz} \leq 1$, we establish that the conditional density $\Pr(X_i^k = x | X_i^k \geq \max_{l \neq k} X_i^l)$ is log-concave by employing the fact that log-concavity is preserved under integration, which has been shown in [Prékopa \(1973\)](#). Then, like in the case of $K = 1$, log-concavity implies the desired Lipschitz condition on $\mu_a^K(z)$. Next, we show that $\frac{d\mu_r^K(z)}{dz} \leq 1$ by directly invoking the log-concavity of f as well as its implications. Again, the preservation of log-concavity under integration due to [Prékopa \(1973\)](#) is important. Taking both aspects together, Lemma 1 follows, and we obtain that the right-hand side of the equation above is decreasing in z . When comparing the welfare induced by single-option sequential search and multi-option sequential search, we repeatedly make use of Lemma 1. We believe that the technical property established in Lemma 1 might be useful beyond its application in this paper.

5 Unanimity Voting

Having established equilibrium existence and uniqueness, in this section we assume that the committee employs unanimity voting, i.e., we set $M = N$. We compare single-option sequential search and different forms of multi-option sequential search in terms of acceptance standards and welfare and show how the superiority of different search technologies depends on the search cost structure. Moreover, for sufficiently small magnitudes of search costs, we derive the welfare-maximizing sample size, depending on the shape of the search cost function. In particular, we identify conditions on the search cost function under which single-option sequential search is optimal and suboptimal respectively.

Consider multi-option sequential search with $K' \geq 1$ and $K \geq 1$ candidates per period, and assume that $K' > K$.

To begin with, as a first cost regime, we study cost functions h that satisfy $\frac{h(K')}{K'} \geq \frac{h(K)}{K}$. This restriction on the function h means that the search costs per candidate when there are K' candidates per period are at least as high as under

the search technology featuring K candidates per period. For instance, this condition is met if $h(K') = (K')^\alpha$ and $h(K) = (K)^\alpha$ for some $\alpha \geq 1$.

Denote the ex ante utilitarian welfare per committee member in the game with K' and K candidates per period by $v_{K'}$ and v_K respectively. Proposition 3 establishes that the welfare under multi-option sequential search with K' candidates per period is strictly lower than the welfare when there are K candidates per period.

Proposition 3. *Suppose that the voting rule is unanimity, i.e., $M = N$, and consider any $K', K \geq 1$ with $K' > K$.*

If the function h satisfies

$$\frac{h(K')}{K'} \geq \frac{h(K)}{K},$$

the committee's ex ante utilitarian welfare is higher under multi-option sequential search with K candidates per period relative to multi-option sequential search with K' candidates per period, i.e., $v_K > v_{K'}$.

In words, under unanimity voting, weakly higher search costs per candidate when the sample size is larger imply that the welfare of the search procedure featuring a larger sample size is strictly lower. This conclusion holds for all magnitudes of search costs as quantified by the parameter c . Moreover, the result does not require the density of the value distribution f to be log-concave²³ and it applies to all equilibria of the discussed search procedures in case a search technology admits more than one equilibrium.

The basic trade-off when moving from K to K' candidates per period is that, on the one hand, the expected value conditional on stopping rises, but on the other hand, expected search costs rise, too. The former effect arises because unanimity voting means that, conditional on stopping, all members vote in favor of the hired candidate, and, when there K' instead of K candidates per period, members only approve some candidate if the associated value is the maximum out of the K' instead of the K values they observe. The latter effect is due to two aspects: First, the probability of hiring an arbitrary candidate k is smaller if K' versus K candidates are reviewed simultaneously, and, second, the search costs per candidate

²³We thank an anonymous referee for pointing this out.

are weakly higher if there are K' compared to K candidates per period. Thus, a priori, the ranking of the two search procedures in terms of welfare is ambiguous. The key proof step is to show that the increase in the expected value conditional on stopping is limited when moving from multi-option sequential search with K to K' candidates per period. This aspect is captured in Lemma 2.

Lemma 2. *Consider any $K', K \geq 1$ with $K' > K$. For all $z_K, z_{K'} \in [0, \bar{x})$ such that $z_K \leq z_{K'}$, it holds*

$$\frac{\mu_a^{K'}(z_{K'}) - z_{K'}}{\mu_a^K(z_K) - z_K} < \frac{\frac{1}{K}[1 - F(z_K)^K]}{\frac{1}{K'}[1 - F(z_{K'})^{K'}]}.$$

Take any possibly non-equilibrium cutoffs $z_K, z_{K'} \in [0, \bar{x})$ such that $z_K \leq z_{K'}$, and consider the ratio of the expected values conditional on stopping net of a cutoff when there are K' candidates and the cutoff is $z_{K'}$ versus having K candidates and the cutoff being z_K . Lemma 2 reveals that an upper bound of this ratio is given by the ratio of the probability that an individual member votes in favor of a candidate k if there K candidates and the cutoff is z_K to this probability if there are K' candidates and the cutoff is $z_{K'}$. We believe that this technical property might be useful beyond its application in this paper.

Now, let us sketch the proof of Proposition 3 for interior cutoffs. In this case, acceptance standards coincide with welfare.²⁴ Consider the ratio of the expected value conditional on stopping net of the cutoff when there are K' candidates compared to the net value when there are K candidates, that is,

$$\frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'}] - z_{K'}}{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K] - z_K},$$

where $z_{K'}$ and z_K denote equilibrium cutoffs when there are K' and K candidates, respectively. Towards a contradiction, assume that $z_K \leq z_{K'}$. By the equilibrium equation, i.e., equation (1), the considered ratio is equal to the ratio of the expected search costs when there are K' versus K candidates. Then, the assumption $\frac{h(K')}{K'} \geq \frac{h(K)}{K}$ on the search cost function yields a lower bound on this ratio of expected search costs. Moreover, while invoking $z_K \leq z_{K'}$ and applying Lemma 2,

²⁴We emphasize that the result also holds if some equilibria constitute boundary solutions.

we obtain an upper bound on the discussed ratio of expected values conditional on stopping. It turns out that the derived lower bound is larger than the upper bound, which constitutes the desired contradiction.

Recall that $K' > K$. Let us turn now to the second cost regime and focus on cost functions h that satisfy $\frac{h(K')}{K'} < \frac{h(K)}{K}$. This assumption is reasonable if there are fixed costs associated with the hiring process or if there are cost savings when multiple candidates can be considered. For example, it is satisfied if $h(K') = (K')^\beta$ and $h(K) = (K)^\beta$ for some $\beta < 1$. Proposition 4 reveals that under this assumption on the search costs, the conclusion of the previous part of this section is partly reversed: If the magnitude of the search costs as quantified by the parameter c is sufficiently small, evaluating K' candidates at a time improves welfare relative to reviewing K candidates at a time.

Proposition 4. *Suppose that the voting rule is unanimity, i.e., $M = N$, assume that the density f is log-concave, and consider any $K', K \geq 1$ with $K' > K$.*

If the function h satisfies

$$\frac{h(K')}{K'} < \frac{h(K)}{K}$$

there exists $\bar{c}_{K',K} > 0$ such that for all $c < \bar{c}_{K',K}$, the committee's ex ante utilitarian welfare is higher under multi-option sequential search with K' candidates per period relative to multi-option sequential search with K candidates per period, i.e., $v_{K'} > v_K$.

To verbalize Proposition 4, under unanimity voting, strictly lower search costs per candidate if the sample size is larger imply that the welfare of the search technology with a larger sample size is strictly higher as long as the magnitude of search costs is sufficiently low.

Intuitively, again, the expected value conditional on stopping is not lower when there are K' relative to K candidates at a time. However, in contrast to the previous cost regime, here, for sufficiently small magnitudes of search costs c , the expected search costs are actually lower if there K' compared to K candidates per period, yielding a higher welfare for the committee if K' instead of K candidates

are evaluated simultaneously in every period of time.

Let us sketch the proof of Proposition 4 in more detail. Assume, by contradiction, that for all $\bar{c}_{K',K} > 0$, there exists $c < \bar{c}_{K',K}$ such that $v_K \geq v_{K'}$. Without loss of generality, suppose that both cutoffs are interior. Then, they coincide with welfare and, thus, we have that $z_K \geq z_{K'}$. First, we show that given $z_K \geq z_{K'}$, the expected value conditional on stopping is increasing when moving from K to K' candidates per period. This is a consequence of the log-concavity of f and, more precisely, the Lipschitz condition $\frac{d\mu_a^{K'}(z)}{dz} \leq 1$ we derived in Lemma 1. The equilibrium condition (1) then implies that the expected search costs have to be higher if K' compared to K candidates are evaluated simultaneously. However, if c becomes small, under both search procedures, the equilibrium acceptance standards are close to the upper bound of the support of the value distribution, $\bar{x}$. This conclusion crucially relies on the fact that the voting rule is unanimity and fails in the case of qualified majority rules distinct from unanimity. Then, even though the probability of hiring an arbitrary candidate k is higher for K than for K' , this probability is small for K and for K' . In fact, if c is small enough, the difference is low enough such that, given $\frac{h(K')}{K'} < \frac{h(K)}{K}$, the expected search costs are overall actually smaller for K' than for K candidates at a time. This is the desired contradiction.

Finally, Propositions 3 and 4 allow us to characterize the welfare-maximizing sample size if the magnitude of search costs is sufficiently small. First, if $\frac{h(K')}{K'} \geq h(1)$ for all $K' > 1$, meaning, the search costs per candidate are minimal if one candidate is evaluated at a time, it is immediate from Proposition 3 that single-option sequential search is optimal for all magnitudes of costs as measured by the parameter c . This finding is stated as Corollary 1.

Corollary 1. *Suppose that the voting rule is unanimity, i.e., $M = N$. If the function h satisfies $\frac{h(K')}{K'} \geq h(1)$ for all $K' > 1$, the committee's ex ante utilitarian welfare is higher under single-option sequential search relative to any form of multi-option sequential search, i.e., $v_1 > v_{K'}$ for all $K' > 1$.*

In contrast, if the search costs per candidate are minimal for some form of multi-option sequential search distinct from single-option sequential search and

for exogenous reasons at most $\bar{K} < \infty$ candidates can be reviewed simultaneously, Propositions 3 and 4 together imply that multi-option sequential search with a sample size that coincides with the smallest minimizer of the search costs per candidate is welfare-maximizing as long as the magnitude of the search costs c is sufficiently small. Corollary 2 captures this result.

Corollary 2. *Suppose that the voting rule is unanimity, i.e., $M = N$, assume that the density f is log-concave, and impose that $h(1) > \min_{1 \leq K \leq \bar{K}} \frac{h(K)}{K}$ for some $1 < \bar{K} < \infty$. Consider the smallest $1 < K' \leq \bar{K}$ such that*

$$\frac{h(K')}{K'} = \min_{1 \leq K \leq \bar{K}} \frac{h(K)}{K}.$$

There exists $\bar{c} > 0$ such that for all $c < \bar{c}$, the committee's ex ante utilitarian welfare is higher under multi-option sequential search with K' candidates per period relative to single-option or any other form of multi-option sequential search featuring at most $\bar{K}$ candidates at a time, i.e., $v_{K'} > v_K$ for all $1 \leq K \leq \bar{K}$ such that $K \neq K'$.

Overall, we conclude that the ranking of single-option sequential search and different forms of multi-option sequential search in terms of welfare as well as the welfare-maximizing number of candidates per period is mainly determined by the shape of the search cost function.

6 Qualified Majority Voting

Having studied the case of unanimity voting, in this section, we turn to qualified majority voting, considering a majority requirement M such that $M < N$. We compare the unique equilibria of different forms of multi-option sequential search in terms of acceptance standards and welfare, and, again, we derive the welfare-maximizing number of candidates per period for small magnitudes of search costs.

As before, consider multi-option sequential search with $K' \geq 1$ and $K \geq 1$ candidates per period, and suppose that $K' > K$. Again, let $v_{K'}$ and v_K be the ex ante utilitarian welfare per committee member if there are K' and K

candidates per period respectively. As already stated, the welfare induced by a search procedure is determined by two ingredients: The expected value conditional on hiring and the expected search costs. To start, we compare in Lemma 3 the expected values conditional on stopping when there are K' versus K candidates per period. Recall that $S^{K'}(z, N, M)$ and $S^K(z, N, M)$ denote the expected value conditional on hiring if there are K' and K candidates at a time respectively.

Lemma 3. *Consider any $K', K \geq 1$ with $K' > K$. For all $z \in [0, \bar{x})$, it holds*

$$S^K(z, N, M) < S^{K'}(z, N, M).$$

Lemma 3 reveals that, when fixing a cutoff value z , the expected value conditional on stopping when the sample size is K' is higher than the corresponding object when the sample size is K . If the voting rule is unanimity, this conclusion is immediate because, in this case, $K' > K$ directly yields

$$\begin{aligned} S^K(z, N, N) &= \mathbb{E}[X_i^k | X_i^k \geq z \text{ and } X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l] \\ &< \mathbb{E}[X_i^k | X_i^k \geq z \text{ and } X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l] = S^{K'}(z, N, N). \end{aligned}$$

Yet, if the voting rule is qualified majority, the conclusion is not obvious because there are two forces pulling in opposite directions. Consider the average representations of the expected values conditional on hiring for both discussed search technologies as introduced in equation (2):

$$\begin{aligned} S^{K'}(z, N, N) &= w^{K'}(z)\mu_a^{K'}(z) + [1 - w^{K'}(z)]\mu_r^{K'}(z) \text{ and} \\ S^K(z, N, N) &= w^K(z)\mu_a^K(z) + [1 - w^K(z)]\mu_r^K(z). \end{aligned}$$

Note that for $M < N$, in contrast to unanimity, it does neither hold that $w^{K'}(z) = 1$ nor $w^K(z) = 1$, but these objects depend non-trivially on the number of candidates per period. Fix a potentially non-equilibrium cutoff value z . Observe that $\mu_a^{K'}(z) > \mu_a^K(z)$ as well as $\mu_r^{K'}(z) > \mu_r^K(z)$, that is, both the expected value conditional on approving as well as conditional on rejecting an arbitrary candidate k are higher if there are K' versus K candidates at a time. Similar to the case

of unanimity voting, $\mu_a^{K'}(z) > \mu_a^K(z)$ holds since a member approves a candidate only if the candidate's value is the highest among the K' versus K values that this member observes. Further, the intuition behind $\mu_r^{K'}(z) > \mu_r^K(z)$ is as follows: If the value of some candidate is above the cutoff z , but some member does not vote in favor of this candidate, this means that this candidate's value is not the maximum out of the K' versus K values this member observes, implying that the considered expected value is lower in the latter case. However, at the same time, we have that $w^{K'}(z) < w^K(z)$: Conditional on stopping, the expected share of members who approve some candidate k decreases when increasing the sample size from K to K' . This holds because the probability that a single member approves a candidate k decreases when moving from K to K' , since the candidate's value has to be the maximum out of K' instead of K values in addition to being above the cutoff z . Finally, since $\mu_a^{K'}(z) > \mu_r^{K'}(z)$ as well as $\mu_a^K(z) > \mu_r^K(z)$, the overall effect on the expected value conditional on stopping is a priori ambiguous. We prove Lemma 3 by employing a technical result from Albrecht et al. (2010) related to the expected share of members who approve some candidate k conditional on stopping. In Proposition 5, we claim that multi-option sequential search with more candidates at a time increases welfare independently of the shape of the cost function as long as the magnitude of the search costs is sufficiently small.

Proposition 5. *Suppose that the voting rule is qualified majority distinct from unanimity, i.e., $M < N$, assume that the density f is log-concave, and consider any $K', K \geq 1$ with $K' > K$.*

There exists $\bar{c}_{K',K} > 0$ such that for all $c < \bar{c}_{K',K}$, the committee's ex ante utilitarian welfare is higher under multi-option sequential search with K' candidates per period relative to multi-option sequential search with K candidates per period, i.e., $v_{K'} > v_K$.

Intuitively, the increase in the expected value conditional on hiring when increasing the sample size from K to K' as revealed by Lemma 3 outweighs the potential rise of expected search costs²⁵ if the magnitude of the search costs c is

²⁵We write potential rise of expected search costs because depending on the shape of the function h the expected search costs might also be lower if there K' versus K candidates in each period of time. Of course, if that is the case, this only reinforces our reasoning.

sufficiently small. We emphasize once again that this result does not depend on the form of the cost function. For any function h , there are cost levels c such that evaluating K candidates at a time is dominated by reviewing K' candidates in each period of time.²⁶

Let us discuss the proof of Proposition 5. To the contrary, suppose that for all $\bar{c}_{K',K} > 0$, there exists $c < \bar{c}_{K',K}$ such that $v_K \geq v_{K'}$. Again, without loss of generality, focus on interior cutoffs. Thus, we have that $z_K \geq z_{K'}$ where, again, z_K and $z_{K'}$ denote the equilibrium cutoffs if there are K and K' candidates per period respectively. Recall Lemma 1: The log-concavity of f is sufficient for $\frac{dS^K(z, N, M)}{dz} \leq 1$. When employing this Lipschitz condition, we obtain that the difference $S^{K'}(z_K, N, M) - S^K(z_K, N, M)$ is bounded above by the difference in expected search costs between the search procedures involving K' and K candidates at a time. Now, in contrast to unanimity voting, if $M < N$, the equilibrium cutoffs arising under both discussed search technologies do not converge to the upper bound of the support of the value distribution as the magnitude of the search costs c becomes small, but they remain bounded away from $\bar{x}$. For the case of single-option sequential search, this observation has been made previously in Albrecht et al. (2010) as well as Compte and Jehiel (2010). The intuition for this result is as follows: Under qualified majority voting, conditional on stopping, a candidate might be hired even though some member did not vote in favor of this candidate. Taking that scenario, which does not arise under unanimity voting, into account, members do not become arbitrarily picky if search costs become small. Consequently, if c goes to 0, the difference in expected search costs discussed above vanishes. However, due to Lemma 3, the difference $S^{K'}(z_K, N, M) - S^K(z_K, N, M)$ remains strictly positive.²⁷ This is the desired contradiction.

Moreover, Proposition 5 allows us to characterize the welfare-maximizing sample size per period for small magnitudes of search costs. Suppose that for exogenous reasons at most $\bar{K} < \infty$ candidates can be reviewed simultaneously in each period of time. Then, Proposition 5 implies the following: Whatever the shape of the

²⁶However, as indicated in Proposition 5, the threshold $\bar{c}_{K',K}$ depends on the precise values of K' and K as well as on the shape of the function h .

²⁷This step fails if the voting rule is unanimity because, in this case, if c goes to 0, z_K converges to $\bar{x}$ and, thus, the difference $S^{K'}(z_K, N, N) - S^K(z_K, N, N)$ would vanish as well.

cost function h , multi-option sequential search with $\bar{K}$ candidates per period is welfare-maximizing as long as the magnitude of the search costs c is sufficiently small. Corollary 3 records this implication.

Corollary 3. *Suppose that the voting rule is qualified majority distinct from unanimity, i.e., $M < N$, assume that the density f is log-concave, and consider any $1 < \bar{K} < \infty$.*

There exists $\bar{c} > 0$ such that for all $c < \bar{c}$, the committee's ex ante utilitarian welfare is higher under multi-option sequential search with $\bar{K}$ candidates per period relative to single-option or any other form of multi-option sequential search featuring at most $\bar{K}$ candidates at a time, i.e., $v_{\bar{K}} > v_K$ for all $1 \leq K < \bar{K}$.

Our analysis reveals that the ranking of different forms of multi-option sequential search as well as the welfare-maximizing number of candidates per period for the single-searcher case do not generally extend to the committee search case. Again, note that the single decision-maker case is equivalent to the case of a committee with size $N = 1$ operating under the unanimity voting rule. Thus, our results from Section 5 apply to the single-agent case. To emphasize the drastically different findings, again, suppose that for exogenous reasons at most $\bar{K} < \infty$ candidates can be evaluated simultaneously in every period. If the function h satisfies $\frac{h(K')}{K'} \geq h(1)$ for all $K' > 1$ and the magnitude of search costs c is small, single-option sequential search is welfare-maximizing under unanimity voting, whereas multi-option sequential search featuring $\bar{K}$ candidates at a time is optimal under qualified majority voting. What drives these considerably different conclusions? Again, consider multi-option sequential search with $K' \geq 1$ and $K \geq 1$ candidates per period, and assume that $K' > K$. If the voting rule is unanimity, there is a race between the difference in the expected values conditional on stopping and the difference in the expected search costs between the search technologies involving K' and K candidates at a time: If c becomes small, the difference in expected search costs between K' and K vanishes, and, in addition, the difference in the expected values conditional on hiring also goes to 0. In contrast, under qualified majority voting, if c becomes small, as in the unanimity voting case, the difference in the expected search costs goes to 0. However, in contrast to the unanimity voting case, the difference in the expected values conditional on stopping does not vanish

because equilibrium cutoffs do not converge to the upper bound of the support of the value distribution, but they stay bounded away from $\bar{x}$. This discrepancy explains why the ranking of the two types of search procedures is different when the voting rule is qualified majority instead of unanimity. Therefore, when comparing the single-searcher case with the committee search case, the choice of the voting rule crucially matters.

7 Extensions and Discussion

In the main model, the committee members' preferences feature independent private values. For the case of unanimity voting, we provide extensions to interdependent as well as correlated values. Moreover, we briefly discuss other voting rules.

7.1 Interdependent and Correlated Values

For the unanimity voting rule, we explore the robustness of our results via two extensions: Allowing for interdependent values instead of private values, and allowing for correlated values instead of independent values.²⁸

First, regarding interdependent values, we follow the approach in [Moldovanu and Shi \(2013\)](#), assuming that the value a member derives from hiring some candidate is a weighted average of his or her own observed signal and the signals of all other members. We establish that the ranking of the acceptance standards implied by different search technologies carries over from the analysis under private values. As far as welfare is concerned, note that under the assumption of interdependent values, acceptance standards and welfare no longer coincide even if the equilibrium cutoff is interior (cf. [Moldovanu and Shi \(2013\)](#)). We find that Proposition 4 extends from the private-values case to interdependent values. Overall, this suggests that our results concerning unanimity voting are not driven by the private-values assumption on preferences.

Second, to relax the assumption that candidates' values are distributed independently across committee members, we introduce an unknown state of the world

²⁸The arguments for these extensions are available on request from the authors.

s_k for each candidate $k \in \mathcal{K}$, which we assume to be independently and identically distributed across time and candidates. Conditional on the state realization s_k , the values associated with candidate k are then independently and identically distributed across committee members. The state-dependent value distributions are assumed to be stochastically ranked according to the likelihood-ratio ordering. While relaxing the independence of values across members, we maintain the assumption that committee members' preferences feature private values. Thus, acceptance standards and welfare again coincide whenever the equilibrium is interior. We find that all results for the unanimity voting rule carry over from the private-values case to correlated values. Therefore, we conclude that, while the assumption of independently distributed values is admittedly strong, it does not drive our results for the unanimity voting rule.

7.2 Other Voting Rules

Let us discuss the class of simple voting rules on which we focus. Recall that each member may either vote for one of the available candidates or may opt to continue search, and a candidate is hired if and only if the number of votes he or she receives exceeds some threshold. Again, as argued in the introduction, considering these voting rules is a natural choice when adopting an approach that is positive with regard to the voting rule, but normative with respect to the search technology. That being said, since members have to decide about more than two alternatives under multi-option sequential search, other voting rules are also conceivable. Inspired by approval voting,²⁹ one might allow the members to approve any number of candidates instead of only one candidate, and assume that, subject to some tie-breaking rule, a candidate is hired if and only if he or she is approved by more members than any other candidate and the number of supporters of this candidate exceeds some threshold. However, the analysis of the equilibrium voting behavior under these approval-based voting rules is much more complicated. Suppose that there are two candidates per period, i.e., $K = 2$, and assume that the mentioned threshold coincides with unanimity, meaning, it

²⁹For an overview about several aspects related to approval voting, we refer to [Laslier and Sanver \(2010\)](#).

equals the committee size N . Even in this simple case, for instance, the strategy “approve all candidates above some cutoff” does not constitute an equilibrium: Whether approving the second-best candidate is beneficial for a member does not only depend on the aspect whether the value of this candidate is above or below the cutoff, but it also matters how much the value of this candidate falls below the value of the best candidate and how much it exceeds the cutoff or continuation value. If the values of the two candidates are both above the cutoff and they are very close to each other, members might want to approve both candidates. In contrast, if the two values are above the cutoff, the value of the best candidate is close to the upper bound of the support of the value distribution, but the value of the second-best candidate is only slightly above the continuation value, members might want to approve only their best candidate. This discussion reveals that already the analysis of the equilibrium voting behavior under these alternative voting rules is rather involved. Consequently, studying the ranking of the search procedures in terms of welfare—which is the focus of this paper—under these alternative voting rules does not seem to be tractable.

8 Conclusion

In this paper, we contrast the well-known sequential search procedure, in which candidates are evaluated “one at a time”, and different forms of multi-option sequential search, in which, in each period, committees simultaneously evaluate a set of candidates of fixed size. We study the equilibrium behavior under these search procedures and show equilibrium existence as well as equilibrium uniqueness within some reasonably restricted class of equilibria. Based on the equilibrium analysis, we compare single-option and various types of multi-option sequential search in terms of acceptance standards and welfare. We identify circumstances under which the “one at a time” policy commonly studied in the committee search literature is not optimal. Generally, the superiority of one or the other search technology depends on two important ingredients of the search problem: The voting rule and the specification of the search costs associated with the simultaneous evaluation of multiple candidates.

If the committee operates under the unanimity rule, the comparison of different search protocols is sensitive to the shape of the cost function. This dependence on the form of the cost function partly vanishes when committees employ a qualified majority rule different from unanimity. In this case, evaluating more candidates at a time improves welfare for any type of cost function as long as the magnitude of the search costs is sufficiently small. Consequently, the assessment of the studied search procedures as well as the underlying trade-offs considerably change when moving from the unanimity rule to qualified majority rules. This is the main qualitative insight of this paper. Again, note that search conducted by a single agent is a special case of committee search with unanimity voting. Consequently, our analysis reveals that the results for the single decision-maker case (see e.g. [Manning and Morgan \(1985\)](#)) do not carry over to the committee setting, but the presence of a committee alters the search design problem and implies different rankings of search procedures.

Appendix A Proofs

A.1 Characterization

To begin with, we claim that the best response of any member $i \in \mathcal{N}$ against an arbitrary neutral stationary Markov strategy that is symmetric across all other members amounts to a maximum-strategy with cutoff, that is, member i votes in favor of candidate $k \in \mathcal{K}$ if and only if

$$x_i^k \geq \max_{l \neq k} x_i^l \text{ and } x_i^k \geq z$$

with $z \in [0, \bar{x})$ being some cutoff.

Assume that all members except for member $i \in \mathcal{N}$ in some period t behave according to a common Markovian strategy that is stationary and neutral. First of all, let v be the continuation value member i obtains when search continues. Note that v does not depend on past or current actions or value realizations since the continuation strategy adopted by all members in periods following t is Markovian. Also, it is neither sensitive to the identity i of the member nor to calendar time because continuation strategies are symmetric across members and stationary. Now, suppose that member i observes the value realizations $(x_i^1, \dots, x_i^K)$ in period t . Member i is pivotal for candidate k if and only if exactly $M - 1$ out of the other $N - 1$ members choose action k in the given period, that is, approve candidate k . Let $p_k(a, b) > 0$ with $a \in \mathbb{N}$, $b \in \mathbb{N}_0$ and $b \leq a$ denote the probability that exactly b out of a members choose action k in the given period. Similarly, $P_k(a, b) > 0$ with $a, b \in \mathbb{N}$ and $b \leq a$ describes the probability that at most $b - 1$ out of a members select action k . Then, the probability that member i is pivotal in favor of candidate k is given by $p_k(N - 1, M - 1)$.

The expected utility that member i obtains when approving candidate k can be expressed as follows:

$$\begin{aligned} & [(1 - P_k(N - 1, M)) + p_k(N - 1, M - 1)][x_i^k] + \sum_{l \in \{1, \dots, K\}: l \neq k} [1 - P_l(N - 1, M)][x_i^l] \\ & + [P_k(N - 1, M) - p_k(N - 1, M - 1) - \sum_{l \in \{1, \dots, K\}: l \neq k} (1 - P_l(N - 1, M))][v]. \end{aligned}$$

The expected payoff of member i when voting in favor of continuing search, i.e., selecting action 0, amounts to

$$\sum_{l \in \{1, \dots, K\}} [1 - P_l(N - 1, M)][x_i^l] + [1 - \sum_{l \in \{1, \dots, K\}} (1 - P_l(N - 1, M))][v].$$

Since the stationary Markov strategy that is commonly adopted by members distinct from i is neutral, it holds that $P_d(a, b) = P_e(a, b)$ as well as $p_d(a, b) = p_e(a, b)$ for all $d, e \in \mathcal{K}$. For simplicity, write $P(a, b)$ and $p(a, b)$ to denote these probabilities. Consequently, the expected utility of choosing action k can be reformulated in the following way:

$$\begin{aligned} p(N - 1, M - 1)[x_i^k] + [1 - P(N - 1, M)][\sum_{l \in \{1, \dots, K\}} x_i^l] \\ + [1 - K(1 - P(N - 1, M)) - p(N - 1, M - 1)][v]. \end{aligned}$$

Similarly, the expected payoff of action 0 simplifies to the expression

$$[1 - P(N - 1, M)][\sum_{l \in \{1, \dots, K\}} x_i^l] + [1 - K(1 - P(N - 1, M))][v].$$

Thus, voting in favor of candidate k is optimal for member i if and only if, for all $m \in \mathcal{K}$ with $m \neq k$,

$$\begin{aligned} p(N - 1, M - 1)[x_i^k] + [1 - P(N - 1, M)][\sum_{l \in \{1, \dots, K\}} x_i^l] \\ + [1 - K(1 - P(N - 1, M)) - p(N - 1, M - 1)][v] \\ \geq p(N - 1, M - 1)[x_i^m] + [1 - P(N - 1, M)][\sum_{l \in \{1, \dots, K\}} x_i^l] \\ + [1 - K(1 - P(N - 1, M)) - p(N - 1, M - 1)][v], \end{aligned}$$

and, at the same time,

$$\begin{aligned}
& p(N-1, M-1)[x_i^k] + [1 - P(N-1, M)][\sum_{l \in \{1, \dots, K\}} x_i^l] \\
& + [1 - K(1 - P(N-1, M)) - p(N-1, M-1)][v] \\
& \geq [1 - P(N-1, M)][\sum_{l \in \{1, \dots, K\}} x_i^l] + [1 - K(1 - P(N-1, M))][v].
\end{aligned}$$

The former condition is equivalent to requiring that $x_i^k \geq \max_{l \neq j} x_i^l$. The latter condition reduces to $x_i^k \geq v$. This means that there exists a cutoff value $z_i(t) \in [0, \bar{x})$ such that this condition is met if and only if $x_i^j \geq z_i(t)$. Moreover, the cutoff value solves $z_i(t) = v$ whenever it is interior. Hence, given an arbitrary neutral stationary Markov strategy commonly adopted by all members except for member i in period t , it is optimal for member i to employ a maximum-strategy with cutoff $z_i(t)$ in this period.

In the following, we make use of this claim, and we establish the sufficiency and the necessity part separately.

With regard to necessity, it is immediate from the previous claim that any symmetric stationary Markov equilibrium in neutral strategies must involve a maximum-strategy with cutoff $z \in [0, \bar{x})$ solving $z = v$ whenever being interior, and that this strategy is commonly adopted by all members since, otherwise, at least one member has a profitable deviation. In particular, the cutoffs are neither sensitive to the members' identities nor to calendar time because, by assumption, equilibria are symmetric and stationary. Moreover, the consistency of continuation values and equilibrium strategies implies that v must satisfy

$$\begin{aligned}
v &= -c \cdot h(K) + [1 - K(1 - P(N, M))]v \\
&+ K \cdot [1 - P(N, M)]\mathbb{E}[X_i^k | \text{candidate } k \text{ hired}].
\end{aligned}$$

Rearranging this equation yields

$$v = -\frac{c \cdot h(K)}{K \cdot [1 - P(N, M)]} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}].$$

Therefore, equilibrium cutoffs solve the equation

$$z = -\frac{c \cdot h(K)}{K \cdot [1 - P(N, M)]} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]$$

whenever they are interior. Finally, recall that $P(N, M)$ denotes the probability that at most $M-1$ out of N members approve some candidate k . Thus, when using the notation introduced in the main text, we have that $P(N, M) = Q^K(z, N, M)$. This concludes the proof of the necessity part.

Next, we turn to sufficiency. First of all, observe that strategy profiles in which all members adopt the same maximum-strategy with cutoff $z \in [0, \bar{x})$ are symmetric, neutral, and stationary Markov. Furthermore, as argued in the necessity part of this proof, these strategy profiles give rise to continuation values satisfying

$$v = -\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}].$$

Consequently, it remains to verify that these strategy profiles constitute equilibria.

To this end, consider any strategy with cutoff $z \in [0, \bar{x})$ solving

$$z = v = -\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]$$

whenever the cutoff z is interior. First, by construction, the consistency of continuation values and strategies is fulfilled. Second, if all members apart from member $i \in \mathcal{N}$ in period t adopt the discussed strategy, the claim above implies that it is optimal for member i to follow the same strategy in period t , that is, the maximum-strategy with cutoff $z_i(t)$ solving $z_i(t) = v = z$ whenever it is interior. Now, the one-shot deviation principle implies that no member has a profitable deviation. Thus, the maximum-strategy with cutoff z solving

$$z = v = -\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} + \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}]$$

whenever being interior constitutes an equilibrium. This completes the sufficiency part.

A.2 Existence and Uniqueness

Proof of Proposition 1.

Recall that $S^K(z, N, M) = \mathbb{E}[X_i^k | \text{candidate } k \text{ accepted}]$. Rewriting equation (1) which characterizes equilibrium cutoff values yields

$$\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} = S^K(z, N, M) - z.$$

Suppose that $z = 0$. In this case, the left-hand side amounts to $\frac{c \cdot h(K)}{K \cdot [1 - Q^K(0, N, M)]} = \frac{c \cdot \frac{h(K)}{K}}{1 - Q^K(0, N, M)}$ and the right-hand side reduces to $S^K(0, N, M)$. In contrast, if $z \rightarrow \bar{x}$, the left-hand side goes to ∞ whereas the right-hand side amounts to $S^K(\bar{x}, N, M) - \bar{x} \leq 0$.

Depending on the magnitude of the search costs c , we perform a case distinction:

$$1) \frac{c \cdot \frac{h(K)}{K}}{1 - Q^K(0, N, M)} < S^K(0, N, M)$$

In this case, we observe that the left-hand side is strictly smaller than the right-hand side of the equilibrium equation when evaluating both sides at $z = 0$. In contrast, if z is sufficiently close to $\bar{x}$, the left-hand side is strictly larger than the right-hand side. Moreover, note that both sides of the equation involve functions that are continuous in z . Hence, the intermediate value theorem yields the existence of a cutoff z that solves equation (1).

$$2) \frac{c \cdot \frac{h(K)}{K}}{1 - Q^K(0, N, M)} = S^K(0, N, M)$$

Here, the cutoff $z = 0$ solves the equilibrium equation which means that the maximum-strategy with cutoff $z = 0$ constitutes an equilibrium.

$$3) \frac{c \cdot \frac{h(K)}{K}}{1 - Q^K(0, N, M)} > S^K(0, N, M)$$

In this case, suppose that all members apart from member $i \in \mathcal{N}$ in period t adopt the maximum-strategy with cutoff $z = 0$. In this case, the arguments in Appendix A.1 still apply, and, thus, it is optimal for member i to follow some maximum-strategy with cutoff. However, since $v = -\frac{c \cdot \frac{h(K)}{K}}{1 - Q^K(0, N, M)} + S^K(0, N, M) < 0$ by assumption, the optimal cutoff for member i in the given period is $z = 0$. The reason is that member i wants to stop search as quickly as possible, and the probability of voting in favor of some candidate k is maximized at $z = 0$. Alluding to the one-deviation-principle, this shows that there exists a boundary equilibrium such that the maximum-strategy with cutoff amounting to $z = 0$ forms an

equilibrium. □

Proof of Lemma 1.

We establish that $S_z^K(z, N, M) \leq 1$ which implies that the function $S^K(z, N, M) - z$ is non-increasing in z . Subsequently, again, we make use of the notation

$$\begin{aligned}\mu_a^K(z) &= \mathbb{E}[X_i^k | X_i^k \geq z \text{ and } X_i^k \geq \max_{l \neq k} X_i^l] \text{ and} \\ \mu_r^K(z) &= \mathbb{E}[X_i^k | X_i^k < z \text{ or } X_i^k < \max_{l \neq k} X_i^l].\end{aligned}$$

Then, as shown in Appendix B.1, $S^K(z, N, M)$ can be expressed as

$$S^K(z, N, M) = w^K(z)\mu_a^K(z) + (1 - w^K(z))\mu_r^K(z),$$

where $w^K(z)$ is given by

$$w^K(z) = \sum_{l=M}^N \frac{q^K(z, N, l)}{1 - Q^K(z, N, M)} \frac{l}{N}.$$

Further, to simplify the notation, define

$$1 - R^K(z) := \Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z).$$

First, we obtain that $\frac{dw^K(z)}{dz} \leq 0$.³⁰ Observe that $w^K(z)$ constitutes the average of terms of form $\frac{l}{N}$ with weights

$$w_l^K(z) := \frac{q^K(z, N, l)}{1 - Q^K(z, N, M)}.$$

We claim that, for all $l < l'$, $\frac{w_l^K(z)}{w_{l'}^K(z)}$ is non-decreasing in z . This means that increasing z yields a stochastic decrease according to the likelihood-ratio ordering which, as is well-known, implies a stochastic decrease in terms of first-order stochastic dominance. Hence, exploiting the average structure of $w^K(z)$, when increasing z , the average $w^K(z)$ decreases. In other words, we have $\frac{dw^K(z)}{dz} \leq 0$. In order to see

³⁰The argument yielding $\frac{dw^K(z)}{dz} \leq 0$ is analogous to step 2 in the proof of Lemma 1 in [Albrecht et al. \(2010\)](#).

that $\frac{w_l^K(z)}{w_{l'}^K(z)}$ is increasing in z , note that

$$\frac{w_l^K(z)}{w_{l'}^K(z)} = \frac{\binom{N}{l}}{\binom{N}{l'}} R^K(z)^{l'-l} (1 - R^K(z))^{l-l'},$$

and, therefore, straightforward differentiation yields

$$\frac{d \frac{w_l^K(z)}{w_{l'}^K(z)}}{dz} = \frac{\binom{N}{l}}{\binom{N}{l'}} \frac{dR^K(z)}{dz} (l' - l) R^K(z)^{l'-l-1} (1 - R^K(z))^{l-l'-1}.$$

The derivation in [Appendix B.2](#) reveals that

$$1 - R^K(z) = \frac{1}{K} [1 - F(z)^K].$$

Thus, $\frac{dR^K(z)}{dz} = F(z)^{K-1} f(z) \geq 0$ and we obtain that $\frac{d \frac{w_l^K(z)}{w_{l'}^K(z)}}{dz} \geq 0$ which is the desired claim. Therefore, we conclude that $\frac{dw^K(z)}{dz} \leq 0$.

Second, we show that $\mu_a^K(z) - z$ is non-increasing or, in other words, $\frac{d\mu_a^K(z)}{dz} \leq 1$.

Consider the density

$$\begin{aligned} g^K(x) &:= \Pr(X_i^k = x | X^k \geq \max_{l \neq k} X_i^l) \\ &= \frac{\Pr(X_i^k = x, X_i^k \geq \max_{l \neq k} X_i^l)}{\Pr(X_i^k \geq \max_{l \neq k} X_i^l)} \\ &= \frac{\Pr(X_i^k = x, x \geq \max_{l \neq k} X_i^l)}{\Pr(X_i^k \geq \max_{l \neq k} X_i^l)} \\ &= \frac{\Pr(X_i^k = x) \Pr(x \geq \max_{l \neq k} X_i^l)}{\Pr(X_i^k \geq \max_{l \neq k} X_i^l)} \\ &= K f(x) [F(x)]^{K-1}. \end{aligned}$$

We know from [Prékopa \(1973\)](#) that the log-concavity of the density f implies that the cdf F is also log-concave. Moreover, since the product of log-concave functions must be again log-concave, we obtain that the density g^K is log-concave as well. Therefore, as is well-known, the log-concavity of g^K implies that the random variable $X_i^k | X_i^k \geq \max_{l \neq k} X_i^l$ has the decreasing mean residual life property which

means that $\mu_a^K(z) - z$ is non-increasing.³¹ Thus, we conclude that $\frac{d\mu_a^K(z)}{dz} \leq 1$. Third, we establish that $\frac{d\mu_r^K(z)}{dz} \leq 1$. By the law of total expectation, we obtain

$$\mu = \mathbb{E}[X_i^k] = \mu_a^K(z)[1 - R(z)] + \mu_r^K(z)R(z).$$

Again, in Appendix B.2, we derive that

$$1 - R^K(z) = \frac{1}{K}[1 - F(z)^K].$$

Thus,

$$\mu = \mu_a^K(z)\left[\frac{1}{K}(1 - F(z)^K)\right] + \mu_r^K(z)\left[1 - \frac{1}{K}(1 - F(z)^K)\right].$$

Let G^K be the cdf of the random variable $X_i^k | X_i^k \geq \max_{l \neq k} X_i^l$. Hence, rearranging yields

$$\begin{aligned} \mu_r^K(z) &= \frac{\mu - \mu_a^K(z)\left[\frac{1}{K}(1 - F(z)^K)\right]}{1 - \frac{1}{K}(1 - F(z)^K)} \\ &= \frac{\int_0^{\bar{x}} sf(s)ds - \left[\frac{1}{K}(1 - F(z)^K)\right] \int_z^{\bar{x}} s \frac{g^K(s)}{1 - G^K(z)} ds}{1 - \frac{1}{K}(1 - F(z)^K)} \\ &= \frac{\int_0^{\bar{x}} sf(s)ds - \int_z^{\bar{x}} sf(s)F(s)^{K-1}ds}{1 - \frac{1}{K}(1 - F(z)^K)}. \end{aligned}$$

³¹Bagnoli and Bergstrom (2005) discuss the relationship between log-concave densities and concepts from reliability theory.

Taking the derivative of $\mu_r^K(z)$ with respect to z yields

$$\begin{aligned}
& \frac{d\mu_r^K(z)}{dz} \\
&= \frac{(zf(z)F(z)^{K-1}) \cdot (1 - \frac{1}{K}(1 - F(z)^K))}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\
&\quad - \frac{(\int_0^{\bar{x}} sf(s)ds - \int_z^{\bar{x}} sf(s)F(s)^{K-1}ds) \cdot f(z)F(z)^{K-1}}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\
&\quad \cdot [z(1 - \frac{1}{K}) + z\frac{1}{K}F(z)^K - sF(s)]_0^{\bar{x}} + \int_0^{\bar{x}} F(s)ds + s\frac{1}{K}F(s)^K \Big|_z^{\bar{x}} - \int_z^{\bar{x}} \frac{1}{K}F(s)^K ds \\
&= \frac{f(z)F(z)^{K-1}}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\
&\quad \cdot [z(1 - \frac{1}{K}) + z\frac{1}{K}F(z)^K - \bar{x}(1 - \frac{1}{K}) - z\frac{1}{K}F(z)^K + \int_0^{\bar{x}} F(s)ds - \int_z^{\bar{x}} \frac{1}{K}F(s)^K ds] \\
&= \frac{f(z)F(z)^{K-1}[(z - \bar{x})(1 - \frac{1}{K}) + \int_0^{\bar{x}} F(s)ds - \int_z^{\bar{x}} \frac{1}{K}F(s)^K ds]}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}[\int_0^{\bar{x}} F(s)ds - \int_z^{\bar{x}} [1 - \frac{1}{K}(1 - F(s)^K)]ds]}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}[\int_0^z F(s)ds + \int_z^{\bar{x}} F(s)ds - \int_z^{\bar{x}} [1 - \frac{1}{K}(1 - F(s)^K)]ds]}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\
&= \frac{f(z)F(z)^{K-1}[\int_0^z F(s)ds + \int_z^{\bar{x}} F(s) - [1 - \frac{1}{K}(1 - F(s)^K)]ds]}{[1 - \frac{1}{K}(1 - F(z)^K)]^2}.
\end{aligned}$$

Since we have $\frac{d\mu_r^K(z)}{dz} \Big|_{z=0} = 0 \leq 1$, for the remainder of the proof of $\frac{d\mu_r^K(z)}{dz} \leq 1$, suppose that $z \neq 0$.

Again, due to [Prékopa \(1973\)](#), log-concavity is preserved under integration. Hence, since the density f is log-concave, the cdf $F(z) = \int_0^z f(s)ds$ is also log-concave and, consequently, the left-hand integral $\int_0^z F(s)ds$ must be log-concave as well. By definition of log-concavity, this means that $\int_0^z F(s)ds \leq \frac{F(z)^2}{f(z)}$.³²

³²Again, for a discussion of these kinds of implications, we refer to [Bagnoli and Bergstrom \(2005\)](#).

Moreover, note that, for all $s \in [0, \bar{x}]$,

$$\begin{aligned} \frac{1}{K}(1 - F(s)^K) &= 1 - R^K(s) = \Pr(X^k \geq \max_{l \neq k} X^l \text{ and } X^k \geq s) \\ &\leq \Pr(X^k \geq s) = 1 - F(s). \end{aligned}$$

Thus, we obtain, for all $s \in [0, \bar{x}]$, that

$$F(s) - [1 - \frac{1}{K}(1 - F(s)^K)] \leq 0,$$

and, in particular, it holds that

$$\int_z^{\bar{x}} F(s) - [1 - \frac{1}{K}(1 - F(s)^K)] ds \leq 0.$$

Also, observe that $F(z) - [1 - \frac{1}{K}(1 - F(z)^K)] \leq 0$ is equivalent to

$$\frac{1}{1 - \frac{1}{K}(1 - F(z)^K)} \leq \frac{1}{F(z)}.$$

Employing the derived inequalities yields

$$\begin{aligned} \frac{d\mu_r^K(z)}{dz} &= \frac{f(z)F(z)^{K-1}[\int_0^z F(s)ds + \int_z^{\bar{x}} F(s) - [1 - \frac{1}{K}(1 - F(s)^K)]ds]}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\ &\leq \frac{f(z)F(z)^{K-1} \int_0^z F(s)ds}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\ &\leq \frac{f(z)F(z)^{K-1} \frac{F(z)^2}{f(z)}}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\ &= \frac{F(z)^{K+1}}{[1 - \frac{1}{K}(1 - F(z)^K)]^2} \\ &\leq \frac{F(z)^{K+1}}{F(z)^2} \\ &= F(z)^{K-1} \\ &\leq 1. \end{aligned}$$

Therefore, we conclude that $\frac{d\mu_r^K(z)}{dz} \leq 1$.

Further, note that $\mu_a^K(z) > \mu_r^K(z)$ or, equivalently, $\mu_a^K(z) - \mu_r^K(z) > 0$. Taking

together the three ingredients $\frac{dw^K(z)}{dz} \leq 0$, $\frac{d\mu_a^K(z)}{dz} \leq 1$ and $\frac{d\mu_r^K(z)}{dz} \leq 1$, we have

$$\begin{aligned}
S_z^K(z, N, M) &= \frac{d[w^K(z)\mu_a^K(z) + (1 - w^K(z))\mu_r^K(z)]}{dz} \\
&= \frac{d[w^K(z)[\mu_a^K(z) - \mu_r^K(z)] + \mu_r^K(z)]}{dz} \\
&= \frac{dw^K(z)}{dz}[\mu_a^K(z) - \mu_r^K(z)] + w^K(z)\left[\frac{d\mu_a^K(z)}{dz} - \frac{d\mu_r^K(z)}{dz}\right] + \frac{d\mu_r^K(z)}{dz} \\
&= \frac{dw^K(z)}{dz}(\mu_a^K(z) - \mu_r^K(z)) + w^K(z)\frac{d\mu_a^K(z)}{dz} + [1 - w^K(z)]\frac{d\mu_r^K(z)}{dz} \\
&\leq w^K(z)\frac{d\mu_a^K(z)}{dz} + [1 - w^K(z)]\frac{d\mu_r^K(z)}{dz} \\
&\leq w^K(z) + [1 - w^K(z)] \\
&= 1.
\end{aligned}$$

In conclusion, as desired, we infer that $S_z^K(z, N, M) \leq 1$ which, implies that the function $S^K(z, N, M) - z$ is non-increasing in z . Additionally, the argument reveals that $S_z^K(z, N, M) < 1$ whenever $z \neq 0$ and, thus, $S^K(z, N, M) - z$ is strictly decreasing in z . $\square$

Proof of Proposition 2.

To begin with, by Proposition 1, there exists an equilibrium. Moreover, we know from Lemma 1 that the function $S^K(z, N, M) - z$ is decreasing in z . Next, we show that the function

$$\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]}$$

is increasing in z .

Again, to simplify the notation, define

$$1 - R^K(z) := \Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z).$$

Taking the derivative of the discussed function with respect to z yields

$$\frac{d}{dz} \left[\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} \right] = \frac{c \cdot h(K) \cdot Q_z^K(z, N, M)}{K \cdot [1 - Q^K(z, N, M)]^2}.$$

Further, using the relationship between the Binomial and the Beta distribution,³³ we have

$$\begin{aligned} Q^K(z, N, M) &= \sum_{l=0}^{M-1} \binom{N}{l} (1 - R^K(z))^l \cdot R^K(z)^{N-l} \\ &= \frac{N!}{(N-M)! \cdot (M-1)!} \int_0^{R^K(z)} s^{N-M} (1-s)^{M-1} ds. \end{aligned}$$

Taking the derivative of $Q^K(z, N, M)$ with respect to z yields

$$Q_z^K(z, N, M) = \frac{N!}{(N-M)! \cdot (M-1)!} \frac{dR^K(z)}{dz} R^K(z)^{N-M} (1 - R^K(z))^{M-1}.$$

Again, the derivation in Appendix B.2 reveals that

$$1 - R^K(z) = \frac{1}{K} [1 - F(z)^K].$$

Thus, we have that $\frac{dR^K(z)}{dz} = F(z)^{K-1} f(z) \geq 0$. Hence, we obtain that

$Q_z^K(z, N, M) \geq 0$, yielding the desired inference that

$$\frac{d}{dz} \left[\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} \right] = \frac{c \cdot h(K) \cdot Q_z^K(z, N, M)}{K \cdot [1 - Q^K(z, N, M)]^2} \geq 0.$$

Additionally, the argument shows that this derivative is strictly larger than 0 whenever $z \neq 0$ and, hence, $\frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]}$ is strictly increasing in z .

Consider the equation characterizing equilibrium cutoff values

$$S^K(z, N, M) - z = \frac{c \cdot h(K)}{K \cdot [1 - Q^K(z, N, M)]} = \frac{c \frac{h(K)}{K}}{1 - Q^K(z, N, M)}.$$

Depending on the magnitude of the search costs, we perform a case distinction:

$$1) \frac{c \frac{h(K)}{K}}{1 - Q^K(0, N, M)} < S^K(0, N, M)$$

In this case, all cutoffs associated with equilibrium strategies are interior, satisfying $z \neq 0$. In particular, these cutoffs must solve the equilibrium equation. However, due to Lemma 1, the left-hand side of the discussed equation is strictly decreasing and the right-hand side is strictly increasing. Therefore, both sides of

³³cf. Casella and Berger (2002)

the equation have at most one intersection which establishes uniqueness of equilibrium.

$$2) \frac{c \frac{h(K)}{K}}{1 - Q^K(0, N, M)} \geq S^K(0, N, M)$$

Here, the cutoff $z = 0$ is part of an equilibrium. Either $z = 0$ solves the equilibrium equation or there is a boundary equilibrium involving the cutoff $z = 0$. To the contrary, suppose that there is another equilibrium with some cutoff $z' > 0$. This cutoff must solve the equilibrium equation because it is interior. However, employing the monotonicity properties of the functions involved in the equilibrium equation that are partly derived in Lemma 1, we have

$$\frac{c \frac{h(K)}{K}}{1 - Q^K(z', N, M)} > \frac{c \frac{h(K)}{K}}{1 - Q^K(0, N, M)} \geq S^K(0, N, M) > S^K(z', N, M) - z'.$$

Hence, the cutoff $z' > 0$ cannot be part of an equilibrium which constitutes the desired contradiction. $\square$

A.3 Unanimity Voting

Proof of Lemma 2.

Consider any $K', K \geq 1$ with $K' > K$. Suppose, by contradiction, that there exist some $z_K, z_{K'} \in [0, \bar{x})$ with $z_K \leq z_{K'}$ such that

$$\begin{aligned} \frac{\mu_a^{K'}(z_{K'}) - z_{K'}}{\mu_a^K(z_K) - z_K} &= \frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'}] - z_{K'}}{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K] - z_K} \\ &\geq \frac{\frac{1}{K} [1 - F(z_K)^K]}{\frac{1}{K'} [1 - F(z_{K'})^{K'}]} = \frac{K'}{K} \frac{[1 - F(z_K)^K]}{[1 - F(z_{K'})^{K'}]}. \end{aligned}$$

Rewriting the left-hand side of the inequality yields

$$\begin{aligned} \frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'}] - z_{K'}}{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K] - z_K} &= \frac{\frac{\int_{z_{K'}}^{\bar{x}} f(s) F(s)^{K'-1} ds}{\frac{1}{K'} [1 - F(z_{K'})^{K'}]} - z_{K'}}{\frac{\int_{z_K}^{\bar{x}} f(s) F(s)^{K-1} ds}{\frac{1}{K} [1 - F(z_K)^K]} - z_K} \\ &= \frac{K' [1 - F(z_K)^K]}{K [1 - F(z_{K'})^{K'}]} \frac{\int_{z_{K'}}^{\bar{x}} f(s) F(s)^{K'-1} ds - z_{K'} [\frac{1}{K'} (1 - F(z_{K'})^{K'})]}{\int_{z_K}^{\bar{x}} f(s) F(s)^{K-1} ds - z_K [\frac{1}{K} (1 - F(z_K)^K)]}, \end{aligned}$$

where the first step uses the fact that

$$\Pr(X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'}) = \frac{1}{K'}[1 - F(z_{K'})^{K'}] \text{ and}$$

$$\Pr(X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K) = \frac{1}{K}[1 - F(z_K)^K],$$

which is derived in Appendix B.2.

Thus, we obtain that

$$\frac{\int_{z_{K'}}^{\bar{x}} f(s)F(s)^{K'-1}sd s - z_{K'}[\frac{1}{K'}(1 - F(z_{K'})^{K'})]}{\int_{z_K}^{\bar{x}} f(s)F(s)^{K-1}sd s - z_K[\frac{1}{K}(1 - F(z_K)^K)]} \geq 1. \quad (3)$$

Observe that $\int_{z_K}^{\bar{x}} f(s)F(s)^{K-1}sd s - z_K[\frac{1}{K}(1 - F(z_K)^K)] > 0$ because this inequality is equivalent to $\mu_a^K(z_K) > z_K$. Therefore, inequality (3) is equivalent to

$$\begin{aligned} & \int_{z_{K'}}^{\bar{x}} f(s)F(s)^{K'-1}sd s - z_{K'}[\frac{1}{K'}(1 - F(z_{K'})^{K'})] \\ & \geq \int_{z_K}^{\bar{x}} f(s)F(s)^{K-1}sd s - z_K[\frac{1}{K}(1 - F(z_K)^K)] =: g(z_K). \end{aligned}$$

The right-hand side of this inequality is non-increasing in z_K . To see this, compute the derivative

$$\begin{aligned} g'(z_K) &= -f(z_K)F(z_K)^{K-1}z_K - \frac{1}{K}[1 - F(z_K)^K] + z_KF(z_K)^{K-1}f(z_K) \\ &= -\frac{1}{K}[1 - F(z_K)^K] \leq 0. \end{aligned}$$

Therefore, because of $z_K \leq z_{K'}$, it follows that

$$\begin{aligned} & \int_{z_{K'}}^{\bar{x}} f(s)F(s)^{K'-1}sd s - z_{K'}[\frac{1}{K'}(1 - F(z_{K'})^{K'})] \\ & \geq \int_{z_{K'}}^{\bar{x}} f(s)F(s)^{K-1}sd s - z_{K'}[\frac{1}{K}(1 - F(z_{K'})^K)]. \end{aligned}$$

Rearranging yields

$$\int_{z_{K'}}^{\bar{x}} f(s)s[F(s)^{K'-1} - F(s)^{K-1}]ds \geq z_{K'}[\frac{1}{K'}(1 - F(z_{K'})^{K'}) - \frac{1}{K}(1 - F(z_{K'})^K)].$$

Since $z_{K'} < \bar{x}$ and $K' > K$,

$$\begin{aligned} \int_{z_{K'}}^{\bar{x}} f(s)s[F(s)^{K'-1} - F(s)^{K-1}]ds &< z_{K'} \int_{z_{K'}}^{\bar{x}} f(s)[F(s)^{K'-1} - F(s)^{K-1}]ds \\ &= z_{K'} \left[\frac{1}{K'}(1 - F(z_{K'})^{K'}) - \frac{1}{K}(1 - F(z_{K'})^K) \right]. \end{aligned}$$

Hence, we have that

$$\begin{aligned} &z_{K'} \left[\frac{1}{K'}(1 - F(z_{K'})^{K'}) - \frac{1}{K}(1 - F(z_{K'})^K) \right] \\ &> \int_{z_{K'}}^{\bar{x}} f(s)s[F(s)^{K'-1} - F(s)^{K-1}]ds \\ &\geq z_{K'} \left[\frac{1}{K'}(1 - F(z_{K'})^{K'}) - \frac{1}{K}(1 - F(z_{K'})^K) \right], \end{aligned}$$

which is the desired contradiction. $\square$

Proof of Proposition 3.

We begin by deriving conditions for when boundary solutions of either of the search procedures arise.

First of all, note that the proof of Proposition 1 reveals that under multi-option sequential search with K candidates per period, there is a boundary equilibrium if and only if

$$c \geq \frac{S^K(0, N, N)[1 - Q^K(0, N, N)]}{\frac{h(K)}{K}} = \frac{\mu_a^K(0)[\frac{1}{K}]^N}{\frac{h(K)}{K}} =: c^K.$$

Similarly, if there are K' candidates per period, a corner solution arises if and only if

$$c \geq \frac{S^{K'}(0, N, N)[1 - Q^{K'}(0, N, N)]}{\frac{h(K')}{K'}} = \frac{\mu_a^{K'}(0)[\frac{1}{K'}]^N}{\frac{h(K')}{K'}} =: c^{K'}.$$

We claim that $c^{K'} < c^K$.

Suppose not, i.e., assume that $c^{K'} \geq c^K$. By definition, this means that

$$\frac{\mu_a^{K'}(0)[\frac{1}{K'}]^N}{\frac{h(K')}{K'}} \geq \frac{\mu_a^K(0)[\frac{1}{K}]^N}{\frac{h(K)}{K}}.$$

Applying Lemma 2 while setting $z_K = z_{K'} = 0$ yields

$$\frac{\mu_a^{K'}(0)}{\mu_a^K(0)} < \frac{K'}{K}.$$

Combining the two inequalities, we obtain

$$\frac{\mu_a^{K'}(0) [\frac{1}{K'}]^N}{\frac{h(K')}{K'}} \geq \frac{\mu_a^K(0) [\frac{1}{K}]^N}{\frac{h(K)}{K}} > \frac{\mu_a^{K'}(0) [\frac{1}{K}]^N}{\frac{h(K)}{K}} \frac{K}{K'}.$$

Hence, since, by assumption, $\frac{h(K')}{K'} \geq \frac{h(K)}{K}$, we have that

$$\frac{\mu_a^{K'}(0) [\frac{1}{K'}]^N}{\frac{h(K')}{K'}} > \frac{\mu_a^{K'}(0) [\frac{1}{K}]^N}{\frac{h(K)}{K}} \frac{K}{K'} \geq \frac{\mu_a^{K'}(0) [\frac{1}{K}]^N}{\frac{h(K')}{K'}} \frac{K}{K'}.$$

Simplifying yields

$$[\frac{1}{K'}]^{N-1} > [\frac{1}{K}]^{N-1}.$$

If $N = 1$, the inequality reduces to $1 > 1$ and, in the case where $N \geq 2$, we must have that $K > K'$. Thus, in both cases, we derived the desired contradiction.

We are now ready to perform a case distinction depending on the magnitude of the scaling parameter c :

1) $c \geq c^K > c^{K'}$

In this case, both search procedures give rise to a unique boundary equilibrium with equilibrium cutoffs $z_K = z_{K'} = 0$. In order to see that there are no additional interior equilibria, consider the search procedure with K' candidates per period. The argument for the search protocol with K candidates at a time is analogous. Towards a contradiction, suppose that there exists an equilibrium with cutoff $z'_{K'} \in (0, \bar{x})$. Since this cutoff is interior, it solves the equilibrium equation

$$c \frac{h(K')}{K'} = [\frac{1}{K'} (1 - F(z'_{K'})^{K'})]^N [\mu_a^{K'}(z'_{K'}) - z'_{K'}].$$

Making use of $z'_{K'} > 0$ and rewriting yield the inequality

$$c \frac{h(K')}{K'} < [\frac{1}{K'}(1 - F(z'_{K'})^{K'})]^N \mu_a^{K'}(z'_{K'}) = [\frac{1}{K'}(1 - F(z'_{K'})^{K'})]^{N-1} \int_{z'_{K'}}^{\bar{x}} f(s) F(s)^{K'-1} s ds.$$

Now, observe that the right-hand side of this inequality is decreasing in the cutoff.

Therefore, it follows that

$$c \frac{h(K')}{K'} < [\frac{1}{K'}]^N \mu_a^{K'}(0),$$

which is equivalent to $c < c^{K'}$. This is the desired contradiction.

The respective welfare levels induced by the unique boundary equilibria of the two search procedures amount to

$$\begin{aligned} v_K &= \mu_a^K(0) - \frac{c \cdot h(K)}{K[\frac{1}{K}]^N} = \mu_a^K(0) - (K)^N c \frac{h(K)}{K} \text{ and} \\ v_{K'} &= \mu_a^{K'}(0) - \frac{c \cdot h(K')}{K'[\frac{1}{K'}]^N} = \mu_a^{K'}(0) - (K')^N c \frac{h(K')}{K'}. \end{aligned}$$

Towards a contradiction, suppose $v_{K'} \geq v_K$. Applying Lemma 2 while setting $z_K = z_{K'} = 0$ and using $\frac{h(K')}{K'} \geq \frac{h(K)}{K}$, we obtain that

$$\begin{aligned} \mu_a^K(0) - (K)^N c \frac{h(K)}{K} = v_K &\leq v_{K'} = \mu_a^{K'}(0) - (K')^N c \frac{h(K')}{K'} \\ &< \mu_a^K(0) \frac{K'}{K} - (K')^N c \frac{h(K)}{K}. \end{aligned}$$

Thus, we conclude that

$$\mu_a^K(0) [\frac{K'}{K} - 1] > c \frac{h(K)}{K} [(K')^N - (K)^N].$$

Since $K' > K$ and $c \geq c^K = \frac{\mu_a^K(0)[\frac{1}{K}]^N}{\frac{h(K)}{K}}$, we have that

$$\mu_a^K(0) [\frac{K'}{K} - 1] > c \frac{h(K)}{K} [(K')^N - (K)^N] \geq \frac{\mu_a^K(0)[\frac{1}{K}]^N}{\frac{h(K)}{K}} \frac{h(K)}{K} [(K')^N - (K)^N].$$

Simplifying yields

$$\frac{K'}{K} > \left(\frac{K'}{K}\right)^N.$$

In the case of $N = 1$, there is a contradiction. If $N \geq 2$, we must have that $K' < K$ which constitutes a contradiction as well.

2) $c^K > c \geq c^{K'}$

Here, if there are K candidates per period, the corresponding search procedure admits only interior equilibria described by cutoff values $z_K > 0$. In contrast, if there are K' candidates per period, as argued above, there is a unique boundary equilibrium with cutoff $z_{K'} = 0$. Therefore, the resulting welfare levels of both search procedures are given by

$$v_K = z_K \text{ and } v_{K'} = \mu_a^{K'}(0) - \frac{c \cdot h(K')}{K' \left[\frac{1}{K'}\right]^N} = \mu_a^{K'}(0) - (K')^N c \frac{h(K')}{K'}.$$

By definition of $c^{K'}$ and because of $c \geq c^{K'}$, we directly obtain that $v_{K'} \leq 0$. In contrast, it holds that $v_K = z_K > 0$, directly implying $v_{K'} < v_K$.

3) $c^K > c^{K'} > c$

In this case, both search technologies give rise to interior equilibria. Denote the equilibrium cutoff values in the game with K' candidates per period by $z_{K'}$ and the cutoff values in the search game with K candidate per period by z_K . Given private value preferences, cutoff values, or acceptance standards, coincide with welfare, i.e., $v_{K'} = z_{K'}$ and $v_K = z_K$.

Assume, by contradiction, that there are equilibria such that $v_K = z_K \leq z_{K'} = v_{K'}$. The equilibrium cutoff values satisfy the following equations:

$$S^K(z_K, N, N) - z_K = \frac{c \cdot h(K)}{K \cdot [1 - Q^K(z_K, N, N)]} \text{ and } S^{K'}(z_{K'}, N, N) - z_{K'} = \frac{c \cdot h(K')}{K' \cdot [1 - Q^{K'}(z_{K'}, N, N)]}.$$

In the following, we derive bounds on the ratio

$$\frac{S^{K'}(z_{K'}, N, N) - z_{K'}}{S^K(z_K, N, N) - z_K} = \frac{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'}] - z_{K'}}{\mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K] - z_K}.$$

First, since $z_K \leq z_{K'}$, Lemma 2 yields

$$\frac{S^{K'}(z_{K'}, N, N) - z_{K'}}{S^K(z_K, N, N) - z_K} < \frac{K'}{K} \frac{[1 - F(z_K)^K]}{[1 - F(z_{K'})^{K'}]}.$$

Second, by the equilibrium conditions, we have that

$$\begin{aligned} \frac{S^{K'}(z_{K'}, N, N) - z_{K'}}{S^K(z_K, N, N) - z_K} &= \frac{\frac{c^{\frac{h(K')}{K'}}}{1 - Q^{K'}(z_{K'}, N, N)}}{\frac{c^{\frac{h(K)}{K}}}{1 - Q^K(z_K, N, N)}} \\ &= \frac{h(K')}{K'} \frac{K}{h(K)} \frac{[\Pr(X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K)]^N}{[\Pr(X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'})]^N}. \end{aligned}$$

Since $\frac{h(K')}{K'} \geq \frac{h(K)}{K}$, we obtain

$$\begin{aligned} &\frac{h(K')}{K'} \frac{K}{h(K)} \frac{[\Pr(X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K)]^N}{[\Pr(X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'})]^N} \\ &\geq \frac{[\Pr(X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K)]^N}{[\Pr(X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'})]^N} \\ &= \left[\frac{K'(1 - F(z_K)^K)}{K(1 - F(z_{K'})^{K'})} \right]^N, \end{aligned}$$

where the last step uses expressions for the involved probabilities that are derived in Appendix B.2. Therefore, we get that

$$\frac{S^{K'}(z_{K'}, N, N) - z_{K'}}{S^K(z_K, N, N) - z_K} \geq \left[\frac{K'(1 - F(z_K)^K)}{K(1 - F(z_{K'})^{K'})} \right]^N.$$

Putting both bounds on $\frac{S^{K'}(z_{K'}, N, N) - z_{K'}}{S^K(z_K, N, N) - z_K}$ together, we conclude that

$$\frac{K'(1 - F(z_K)^K)}{K(1 - F(z_{K'})^{K'})} > \left[\frac{K'(1 - F(z_K)^K)}{K(1 - F(z_{K'})^{K'})} \right]^N.$$

If $N = 1$, there is a contradiction. If $N \geq 2$, this inequality is equivalent to

$$1 > \frac{\frac{1}{K}[1 - F(z_K)^K]}{\frac{1}{K'}[1 - F(z_{K'})^{K'}]}.$$

Because of $z_K \leq z_{K'}$, it follows that

$$1 > \frac{\frac{1}{K}[1 - F(z_K)^K]}{\frac{1}{K'}[1 - F(z_K)^{K'}]}.$$

Now, observe that the term on the right-hand side of this inequality is the ratio of the probabilities of voting in favor of a candidate k when there K compared to K' candidates per period for a fixed cutoff z_K . Since this probability is smaller for K than for K' candidates per period, this ratio must be strictly larger than 1. This is the desired contradiction. Consequently, it must be true that $v_K = z_K > z_{K'} = v_{K'}$. $\square$

Proof of Proposition 4.

To begin with, denote the unique equilibrium cutoff values in the games with K' and K candidates per period by $z_{K'}$ and z_K respectively. To the contrary, suppose that for all $\bar{c}_{K',K} > 0$ there exists $c < \bar{c}_{K',K}$ such that $v_K \geq v_{K'}$. Without loss of generality, restrict attention to sufficiently small values of c such that the equilibria under both procedures are interior. Then, cutoff values coincide with welfare, i.e., $v_K = z_K$ and $v_{K'} = z_{K'}$.

The respective equilibrium thresholds satisfy the following equations:

$$S^K(z_K, N, N) - z_K = \frac{c \cdot h(K)}{K \cdot [1 - Q^K(z_K, N, N)]} \text{ and}$$

$$S^{K'}(z_{K'}, N, N) - z_{K'} = \frac{c \cdot h(K')}{K' \cdot [1 - Q^{K'}(z_{K'}, N, N)]}.$$

Lemma 1 implies that

$$S^{K'}(z_K, N, N) - z_{K'} = \mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'}] - z_{K'}$$

is non-increasing in $z_{K'}$.

Therefore, the assumption $z_K \geq z_{K'}$ yields the inequality

$$\begin{aligned} & \mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_K] - z_K \\ & \leq \mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'}] - z_{K'}. \end{aligned}$$

Moreover, since $K' > K$, it holds that

$$\begin{aligned} & \mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K] \\ & \leq \mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_K]. \end{aligned}$$

Therefore, it follows that

$$\begin{aligned} & \mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K] - z_K \\ & \leq \mathbb{E}[X_i^k | X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'}] - z_{K'}. \end{aligned}$$

This inequality is the same as

$$S^K(z_K, N, N) - z_K \leq S^{K'}(z_{K'}, N, N) - z_{K'}.$$

Exploiting the equilibrium equations, we get that

$$\frac{c^{\frac{h(K)}{K}}}{1 - Q^K(z_K, N, N)} \leq \frac{c^{\frac{h(K')}{K'}}}{1 - Q^{K'}(z_{K'}, N, N)}.$$

Rewriting this inequality yields

$$\begin{aligned} & [\Pr(X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'})]^N \\ & \leq \frac{h(K')}{K'} \frac{K}{h(K)} [\Pr(X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K)]^N. \end{aligned}$$

Furthermore, because of $z_K \geq z_{K'}$, we have that

$$\begin{aligned} & \Pr(X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_K) \\ & \leq \Pr(X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_{K'}). \end{aligned}$$

Thus, we obtain that

$$\begin{aligned} & [\Pr(X_i^k \geq \max_{l \in \{1, \dots, K'\}: l \neq k} X_i^l, X_i^k \geq z_{K'})]^N \\ & \leq \frac{h(K')}{K'} \frac{K}{h(K)} [\Pr(X_i^k \geq \max_{l \in \{1, \dots, K\}: l \neq k} X_i^l, X_i^k \geq z_{K'})]^N. \end{aligned}$$

Rearranging this inequality while employing expressions for the involved probabilities derived in Appendix B.2 implies that

$$\left[\frac{\frac{1}{K'}(1 - F(z_{K'}^{K'}))}{\frac{1}{K}(1 - F(z_{K'}^K))} \right]^N \leq \frac{h(K')}{K'} \frac{K}{h(K)}.$$

Since, by assumption, $\frac{h(K')}{K'} < \frac{h(K)}{K}$, we have that the right-hand side of this inequality is strictly smaller than 1. We claim that, no matter the fixed value of $0 < \frac{h(K')}{K'} \frac{K}{h(K)} < 1$, as long as the cost parameter c is sufficiently small, the left-hand side of the inequality is below 1, but arbitrarily close to it. The first part of this statement is true because the discussed term is the ratio of the probabilities of accepting a candidate k when there K' compared to K candidates per period for a fixed cutoff $z_{K'}$. To see the second part, note that as $c \rightarrow 0$, $z_{K'} \rightarrow \bar{x}$, implying that $F(z_{K'}) \rightarrow 1$. Then, l'Hôpital's rule yields

$$\lim_{c \rightarrow 0} \left[\frac{\frac{1}{K'}(1 - F(z_{K'}^{K'}))}{\frac{1}{K}(1 - F(z_{K'}^K))} \right]^N = \left[\lim_{c \rightarrow 0} \frac{-F(z_{K'}^{K'-1})f(z_{K'})}{-F(z_{K'}^{K-1})f(z_{K'})} \right]^N = \left[\lim_{c \rightarrow 0} F(z_{K'})^{K'-K} \right]^N = 1.$$

Thus, as $c \rightarrow 0$, the left-hand side of the inequality converges to 1. Consequently, eventually, for small c , the left-hand side of the inequality exceeds the right-hand side because $\frac{h(K')}{K'} \frac{K}{h(K)} < 1$. This is the desired contradiction. $\square$

A.4 Qualified Majority Voting

Proof of Lemma 3.

To begin with, take any $K', K \geq 1$ with $K' > K$, and fix any value $z \in [0, \bar{x})$.

In order to improve readability, we often drop the dependence of the involved functions on z . The subsequent argument does not apply to the case in which $K = 1$ and $z = 0$. We tackle this case separately at the end of this proof.

First, we derive an expression for $S^K(z, N, M)$ in terms of $w^K(z)$, $\mu_a^K(z)$, $F(z)$ and μ . By the law of total expectation, we have

$$\mu_r^K = \frac{\mu - \frac{1}{K}(1 - F^K)\mu_a^K}{1 - \frac{1}{K}(1 - F^K)}$$

and, consequently, we obtain

$$\mu_a^K - \mu_r^K = \mu_a^K - \frac{\mu - \frac{1}{K}(1 - F^K)\mu_a^K}{1 - \frac{1}{K}(1 - F^K)} = \frac{\mu_a^K - \mu}{1 - \frac{1}{K}(1 - F^K)}.$$

Therefore, $S^K(z, N, M)$ can be written as

$$\begin{aligned} S^K(z, N, M) &= w^K \mu_a^K + [1 - w^K] \mu_r^K \\ &= \mu_r^K + w^K [\mu_a^K - \mu_r^K] \\ &= \frac{\mu - \frac{1}{K}(1 - F^K)\mu_a^K}{1 - \frac{1}{K}(1 - F^K)} + w^K \frac{\mu_a^K - \mu}{1 - \frac{1}{K}(1 - F^K)} \\ &= \mu \left[\frac{1 - w^K}{1 - \frac{1}{K}(1 - F^K)} \right] + \mu_a^K \left[\frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)} \right] \\ &= \mu + \left[\frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)} \right] [\mu_a^K - \mu]. \end{aligned}$$

Further, the law of total expectation yields

$$S^K(z, N, M) = \frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)} [\mu_a^K - \mu] + \frac{1}{K'} (1 - F^{K'}) \mu_a^{K'} + \left[1 - \frac{1}{K'} (1 - F^{K'}) \right] \mu_r^{K'}.$$

Second, we develop an expression for $\mu_a^{K'} - \mu_r^{K'}$ as well as a lower bound on this term. The law of total expectation implies

$$\mu_r^{K'} = \frac{\mu - \frac{1}{K'}(1 - F^{K'})\mu_a^{K'}}{1 - \frac{1}{K'}(1 - F^{K'})}.$$

Thus, we obtain

$$\begin{aligned}\mu_a^{K'} - \mu_r^{K'} &= \mu_a^{K'} - \frac{\mu - \frac{1}{K'}(1 - F^{K'})\mu_a^{K'}}{1 - \frac{1}{K'}(1 - F^{K'})} \\ &= \frac{\mu_a^{K'} - \mu}{1 - \frac{1}{K'}(1 - F^{K'})} \\ &\geq \frac{\mu_a^K - \mu}{1 - \frac{1}{K'}(1 - F^{K'})},\end{aligned}$$

where the inequality follows from the assumption $K' > K$ which implies $\mu_a^{K'} \geq \mu_a^K$.

Now, suppose to the contrary that $S^K(z, N, M) \geq S^{K'}(z, N, M)$. This means that

$$\begin{aligned}S^K(z, N, M) &= \frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)}[\mu_a^K - \mu] + \frac{1}{K'}(1 - F^{K'})\mu_a^{K'} + [1 - \frac{1}{K'}(1 - F^{K'})]\mu_r^{K'} \\ &\geq \mu_a^{K'}w^{K'} + \mu_r^{K'}[1 - w^{K'}] = S^{K'}(z, N, M).\end{aligned}$$

Rearranging this inequality yields

$$\begin{aligned}&\frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)}[\mu_a^K - \mu] + \mu_r^{K'}[1 - \frac{1}{K'}(1 - F^{K'}) - 1 + w^{K'}] \\ &\geq \mu_a^{K'}[w^{K'} - \frac{1}{K'}(1 - F^{K'})],\end{aligned}$$

which is equivalent to

$$\frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)}[\mu_a^K - \mu] \geq [\mu_a^{K'} - \mu_r^{K'}][w^{K'} - \frac{1}{K'}(1 - F^{K'})].$$

Employing the lower bound on $\mu_a^{K'} - \mu_r^{K'}$, we have

$$\frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)}[\mu_a^K - \mu] \geq \frac{\mu_a^K - \mu}{1 - \frac{1}{K'}(1 - F^{K'})}[w^{K'} - \frac{1}{K'}(1 - F^{K'})]$$

because $w^{K'} - \frac{1}{K'}(1 - F^{K'}) > 0$. To see the latter point, observe that

$$\begin{aligned} w^{K'} &= \sum_{l=M}^N \frac{q^{K'}(z, N, l)}{1 - Q^{K'}(z, N, M)} \frac{l}{N} \\ &\geq \frac{M}{N} \sum_{l=M}^N \frac{q^{K'}(z, N, l)}{1 - Q^{K'}(z, N, M)} = \frac{M}{N} > \frac{1}{2}. \end{aligned}$$

Moreover, since $K' > K \geq 1$, we have

$$\frac{1}{K'}(1 - F^{K'}) \leq \frac{1}{2}(1 - F^{K'}) \leq \frac{1}{2}.$$

Hence, it holds that $w^{K'} - \frac{1}{K'}(1 - F^{K'}) > 0$.

Next, we note that $[\mu_a^K - \mu] > 0$ because F has full support and, by assumption, $z > 0$. Thus, we arrive at the following expression:

$$\frac{w^K - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K}(1 - F^K)} \geq \frac{w^{K'} - \frac{1}{K'}(1 - F^{K'})}{1 - \frac{1}{K'}(1 - F^{K'})}.$$

Rewriting this inequality yields

$$1 - w^K \leq \frac{1 - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K'}(1 - F^{K'})} [1 - w^{K'}]. \quad (4)$$

Now, [Albrecht et al. \(2010\)](#) provide an alternative expression for the weight w^1 as a function of the probability that some member votes in favor of the available candidate. They rely on the Gaussian hypergeometric function as well as the Euler integral.³⁴ We apply those expressions to the weights w^K and $w^{K'}$. In order to simplify the notation, let A^K and $A^{K'}$ be the probability of approving some candidate k if there are K or K' candidates respectively. In other words, define

$$\begin{aligned} A^K(z) &:= \frac{1}{K}(1 - F^K), \text{ as well as} \\ A^{K'}(z) &:= \frac{1}{K'}(1 - F^{K'}). \end{aligned}$$

³⁴See for example [Abramowitz and Stegun \(1965\)](#).

Making use of this notation, the expressions in [Albrecht et al. \(2010\)](#) read as follows:³⁵

$$w^K = A^K + \frac{M}{N}(1 - A^K)\left\{\int_0^1 \left[1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1} \text{ and}$$

$$w^{K'} = A^{K'} + \frac{M}{N}(1 - A^{K'})\left\{\int_0^1 \left[1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}.$$

Therefore, we obtain

$$\begin{aligned} 1 - w^K &= 1 - A^K - \frac{M}{N}(1 - A^K)\left\{\int_0^1 \left[1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1} \\ &= [1 - A^K] \cdot \left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right] \\ &= \left[1 - \frac{1}{K}(1 - F^K)\right] \cdot \left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right], \end{aligned}$$

as well as

$$\begin{aligned} 1 - w^{K'} &= 1 - A^{K'} - \frac{M}{N}(1 - A^{K'})\left\{\int_0^1 \left[1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1} \\ &= [1 - A^{K'}] \cdot \left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right] \\ &= \left[1 - \frac{1}{K'}(1 - F^{K'})\right] \cdot \left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right]. \end{aligned}$$

Then, inequality (4) becomes

$$\begin{aligned} &\left[1 - \frac{1}{K}(1 - F^K)\right] \cdot \left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right] \\ &\leq \frac{1 - \frac{1}{K}(1 - F^K)}{1 - \frac{1}{K'}(1 - F^{K'})} \cdot \left[1 - \frac{1}{K'}(1 - F^{K'})\right] \\ &\quad \cdot \left[1 - \frac{M}{N}\left\{\int_0^1 \left[1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}})\right]^{N-M} dy\right\}^{-1}\right]. \end{aligned}$$

Simplifying and rearranging this inequality yields

$$\int_0^1 \left[1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}})\right]^{N-M} dy \leq \int_0^1 \left[1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}})\right]^{N-M} dy.$$

³⁵The derivation can be found on pages 1403 f. in [Albrecht et al. \(2010\)](#).

In the following, we claim that, for all $y \in [0, 1)$,

$$\left[1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}})\right]^{N-M} > \left[1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}})\right]^{N-M},$$

which implies that the former inequality cannot be true.

To begin with, note that $A^K = A^K(z) > A^{K'}(z) = A^{K'}$ since $z \neq \bar{x}$ and $K' > K$.

Now, take any $y \in [0, 1)$ and observe that

$$\begin{aligned} & A_K > A_{K'} \\ \Leftrightarrow & \frac{A^K}{1 - A^K} > \frac{A^{K'}}{1 - A^{K'}} \\ \Leftrightarrow & 1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}}) > 1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}}) \\ \Leftrightarrow & \left[1 + \frac{A^K}{1 - A^K}(1 - y^{\frac{1}{M}})\right]^{N-M} > \left[1 + \frac{A^{K'}}{1 - A^{K'}}(1 - y^{\frac{1}{M}})\right]^{N-M}. \end{aligned}$$

This establishes the claim, yielding the desired contradiction. Therefore, overall, we conclude that $S^K(z, N, M) < S^{K'}(z, N, M)$ for all $z \in (0, \bar{x})$.

Finally, it remains to tackle the case in which $K = 1$ and $z = 0$. Here, observe that $S^1(0, N, M) = \mu$. Towards a contradiction, suppose that $\mu = S^1(0, N, M) \geq S^{K'}(0, N, M)$. By the law of total expectation, we obtain

$$\begin{aligned} \mu &= \left[\frac{1}{K'}(1 - [F(0)]^{K'})\right]\mu_a^{K'} + \left[1 - \frac{1}{K'}(1 - [F(0)]^{K'})\right]\mu_r^{K'} \\ &\geq S^{K'}(0, N, M) = \mu_a^{K'} w^{K'} + \mu_r^{K'} [1 - w^{K'}]. \end{aligned}$$

Rearranging this inequality yields

$$0 \geq [\mu_a^{K'} - \mu_r^{K'}][w^{K'} - \frac{1}{K'}].$$

However, we have that

$$0 \geq [\mu_a^{K'} - \mu_r^{K'}][w^{K'} - \frac{1}{K'}] > 0$$

because $\mu_a^{K'} - \mu_r^{K'} > 0$ as well as $w^{K'} - \frac{1}{K'}(1 - [F(0)]^{K'}) > 0$. The latter point is implied by $K' > 1$ and it has been established in the first part of this proof.

Hence, we arrive at the desired contradiction. $\square$

Proof of Proposition 5.

Consider any $K', K \geq 1$ with $K' > K$. To the contrary, suppose that for all $\bar{c}_{K',K} > 0$ there exists $c < \bar{c}_{K',K}$ such that $v_K \geq v_{K'}$. Without loss of generality, restrict attention to sufficiently small values of c such that the unique equilibria under both procedures are interior. Let z_K and $z_{K'}$ denote the equilibrium cutoffs corresponding to multi-option sequential search with K and K' candidates per period respectively. These cutoffs solve the respective equilibrium equations

$$\begin{aligned} S^K(z_K, N, M) - z_K &= \frac{c^{\frac{h(K)}{K}}}{1 - Q^K(z_K, N, M)} \\ S^{K'}(z_{K'}, N, M) - z_{K'} &= \frac{c^{\frac{h(K')}{K'}}}{1 - Q^{K'}(z_{K'}, N, M)}, \end{aligned}$$

and they coincide with welfare, meaning, $z_K = v_K$ as well as $z_{K'} = v_{K'}$. Thus, by assumption, $z_K \geq z_{K'}$. Lemma 1 implies that the function $S^K(z, N, M) - z$ is decreasing in z . Making use of this property and employing the equilibrium equations as well as $z_K \geq z_{K'}$, we obtain

$$\begin{aligned} \frac{c^{\frac{h(K)}{K}}}{1 - Q^K(z_K, N, M)} &= S^K(z_K, N, M) - z_K \\ &\leq S^K(z_{K'}, N, M) - z_{K'} \\ &= S^K(z_{K'}, N, M) + \frac{c^{\frac{h(K')}{K'}}}{1 - Q^{K'}(z_{K'}, N, M)} - S^{K'}(z_{K'}, N, M). \end{aligned}$$

Rearranging this inequality yields

$$S^{K'}(z_{K'}, N, M) - S^K(z_{K'}, N, M) \leq \frac{c^{\frac{h(K')}{K'}}}{1 - Q^{K'}(z_{K'}, N, M)} - \frac{c^{\frac{h(K)}{K}}}{1 - Q^K(z_K, N, M)}. \quad (5)$$

Now, we claim that there exists $B < \bar{x}$ such that for all $c > 0$, it holds $z_K < B$ and $z_{K'} < B$.

First, towards a contradiction, suppose that for all $B^K < \bar{x}$ there exist $c > 0$ such that $z_K \geq B^K$. By the equilibrium equation and the monotonicity properties

of the involved functions established in the proofs of Lemma 1 and Proposition 2, we have that z_K is weakly decreasing in c . Thus, the previous assumption requires that $z_K \rightarrow \bar{x}$ as $c \rightarrow 0$. Consider the following rearranged version of the equilibrium equation:

$$z_K = S^K(z_K, N, M) - \frac{c \frac{h(K)}{K}}{1 - Q^K(z_K, N, M)}.$$

If we take the limit on both sides of the equation as $c \rightarrow 0$, we obtain

$$\begin{aligned} \bar{x} &= \lim_{c \rightarrow 0} [z_K] = \lim_{c \rightarrow 0} \left[S^K(z_K, N, M) - \frac{c \frac{h(K)}{K}}{1 - Q^K(z_K, N, M)} \right] \\ &\leq \lim_{c \rightarrow 0} [S^K(z_K, N, M)] < \bar{x}, \end{aligned}$$

which constitutes the desired contradiction. Recalling the average representation of $S^K(z_K, N, M)$, the final inequality holds because $\lim_{c \rightarrow 0} \mu_r^K(z_K) = \mu < \bar{x}$ as well as $\lim_{c \rightarrow 0} w^K(z_K) = \frac{M}{N} < 1$ which is implied by $M < N$. Therefore, there exists $B^K < \bar{x}$ such that for all $c > 0$, it holds that $z_K < B^K$.

Second, applying the same argument in an analogous way to multi-option sequential search with K' candidates per period, we infer that there exists $B^{K'} < \bar{x}$ such that for all $c > 0$, it holds that $z_{K'} < B^{K'}$.

Consequently, setting $B := \max\{B^K, B^{K'}\}$, we conclude that $z_K < B$ and $z_{K'} < B$ for all $c > 0$.

Making use of this feature, we obtain the following upper bound on the right-hand side of inequality (5):

$$\begin{aligned} &\frac{c \frac{h(K')}{K'}}{1 - Q^{K'}(z_{K'}, N, M)} - \frac{c \frac{h(K)}{K}}{1 - Q^K(z_K, N, M)} \\ &< \frac{c \frac{h(K')}{K'}}{1 - Q^{K'}(B, N, M)} - \frac{c \frac{h(K)}{K}}{1 - Q^K(0, N, M)} \\ &= c \left[\frac{\frac{h(K')}{K'}}{1 - Q^{K'}(B, N, M)} - \frac{\frac{h(K)}{K}}{1 - Q^K(0, N, M)} \right]. \end{aligned}$$

Note that this upper bound does not depend on the equilibrium cutoffs of the two considered procedures z_K and $z_{K'}$.

Let us perform a case distinction:

$$1) \frac{\frac{h(K')}{K'}}{1-Q^{K'}(B,N,M)} - \frac{\frac{h(K)}{K}}{1-Q^K(0,N,M)} \leq 0$$

In this case, inequality (5) and the upper bound on the right-hand side of this inequality yield

$$S^{K'}(z_{K'}, N, M) - S^K(z_{K'}, N, M) \leq 0,$$

which contradicts Lemma 3 because of $K' > K$. Let $\bar{c}_{K',K}$ be the cost value such that for all $c < \bar{c}_{K',K}$, the unique equilibrium under both search procedures is interior. That is, set

$$\bar{c}_{K',K} := \min\left\{\frac{S^{K'}(0, N, M)[1 - Q^{K'}(0, N, M)]}{\frac{h(K')}{K'}}, \frac{S^K(0, N, M)[1 - Q^K(0, N, M)]}{\frac{h(K)}{K}}\right\} > 0,$$

recalling the proofs of Propositions 1 and 2. Then, the established contradiction implies that, for all these levels of c , we have $v_K < v_{K'}$.

$$2) \frac{\frac{h(K')}{K'}}{1-Q^{K'}(B,N,M)} - \frac{\frac{h(K)}{K}}{1-Q^K(0,N,M)} > 0$$

To begin with, define

$$r := \min_{s \in [0, B]} [S^{K'}(s, N, M) - S^K(s, N, M)].$$

Observe that r is well-defined because the involved minimum exists due to the extreme value theorem. Further, Lemma 3 implies that $r > 0$. Also, note that r does not depend on z_K , $z_{K'}$ and c . Moreover, we have that the left-hand side of inequality (5) is bounded below by r , meaning,

$$S^{K'}(z_{K'}, N, M) - S^K(z_{K'}, N, M) \geq r.$$

Taking the upper bound on the right-hand side of inequality (5) together with this lower bound on the left hand-side of the discussed inequality, we arrive at the following inequality:

$$r < c \left[\frac{\frac{h(K')}{K'}}{1 - Q^{K'}(B, N, M)} - \frac{\frac{h(K)}{K}}{1 - Q^K(0, N, M)} \right].$$

Now, set

$$\bar{c}_{K',K} := \frac{r}{\frac{\frac{h(K')}{K'}}{1-Q^{K'}(B,N,M)} - \frac{\frac{h(K)}{K}}{1-Q^K(0,N,M)}}.$$

Note that $\bar{c}_{K',K} > 0$ since $\frac{\frac{h(K')}{K'}}{1-Q^{K'}(B,N,M)} - \frac{\frac{h(K)}{K}}{1-Q^K(0,N,M)} > 0$ by assumption and, again, $r > 0$ because of Lemma 3. Then, for all $c < \bar{c}_{K',K}$, we have that

$$\begin{aligned} r &< c \left[\frac{\frac{h(K')}{K'}}{1-Q^{K'}(B,N,M)} - \frac{\frac{h(K)}{K}}{1-Q^K(0,N,M)} \right] \\ &< \frac{r}{\frac{\frac{h(K')}{K'}}{1-Q^{K'}(B,N,M)} - \frac{\frac{h(K)}{K}}{1-Q^K(0,N,M)}} \cdot \left[\frac{\frac{h(K')}{K'}}{1-Q^{K'}(B,N,M)} - \frac{\frac{h(K)}{K}}{1-Q^K(0,N,M)} \right] \\ &= r. \end{aligned}$$

This constitutes the desired contradiction. □

Appendix B Derivations

B.1 Expected Value Conditional on Stopping

First, we derive the expression for the value quality of some candidate $k \in \mathcal{K}$ for some member $i \in \mathcal{N}$ conditional on stopping:

$$\begin{aligned}
S^K(z, N, M) &= \mathbb{E}[X_i^k | \text{candidate } k \text{ hired}] \\
&= \sum_{l=M}^N \Pr(\#k \text{ supporters} = l | k \text{ hired}) \mathbb{E}[X_i^k | k \text{ hired and } \#k \text{ supporters} = l] \\
&= \sum_{l=M}^N \frac{q^K(z, N, l)}{1 - Q^K(z, N, M)} \mathbb{E}[X_i^k | \#k \text{ supporters} = l] \\
&= \sum_{l=M}^N \frac{q^K(z, N, l)}{1 - Q^K(z, N, M)} \cdot \\
&\quad \{ \Pr(\text{voter } i \text{ supports } k | \#k \text{ supporters} = l) \mathbb{E}[X_i^k | \text{voter } i \text{ supports } k] \\
&\quad + \Pr(\text{voter } i \text{ rejects } k | \#k \text{ supporters} = l) \mathbb{E}[X_i^k | \text{voter } i \text{ rejects } k] \} \\
&= \sum_{l=M}^N \frac{q^K(z, N, l)}{1 - Q^K(z, N, M)} \left[\frac{l}{N} \mu_a^K(z) + \frac{N-l}{N} \mu_r^K(z) \right] \\
&= w^K(z) \mu_a^K(z) + [1 - w^K(z)] \mu_r^K(z),
\end{aligned}$$

where $w^K(z)$ is defined as

$$w^K(z) := \sum_{l=M}^N \frac{q^K(z, N, l)}{1 - Q^K(z, N, M)} \frac{l}{N}.$$

B.2 Probability of Acceptance

Second, we derive the expression for the probability that some member $i \in \mathcal{N}$ votes in favor of some candidate $k \in \mathcal{K}$ as a function of K , F and the employed

cutoff z :

$$\begin{aligned}
& \Pr(X_i^k \geq \max_{l \neq k} X_i^l, X_i^k \geq z) \\
&= \int_0^{\bar{x}} \Pr(X_i^k \geq s, X_i^k \geq z) \Pr(\max_{l \neq k} X_i^l = s) ds \\
&= \int_0^{\bar{x}} \Pr(X_i^k \geq \max\{s, z\}) \Pr(\max_{l \neq k} X_i^l = s) ds \\
&= \int_0^z \Pr(X_i^k \geq z) \Pr(\max_{l \neq k} X_i^l = s) ds + \int_z^{\bar{x}} \Pr(X_i^k \geq s) \Pr(\max_{l \neq k} X_i^l = s) ds \\
&= [1 - F(z)] \int_0^z \frac{dF(s)^{K-1}}{ds} ds + \int_z^{\bar{x}} [1 - F(s)] (K-1) F(s)^{K-2} f(s) ds \\
&= [1 - F(z)] F(z)^{K-1} + \int_z^{\bar{x}} (K-1) F(s)^{K-2} f(s) ds \\
&\quad - \int_z^{\bar{x}} (K-1) F(s)^{K-1} f(s) ds \\
&= [1 - F(z)] F(z)^{K-1} + \int_z^{\bar{x}} \frac{dF(s)^{K-1}}{ds} ds - \int_z^{\bar{x}} \frac{d[\frac{K-1}{K} F(s)^K]}{ds} ds \\
&= [1 - F(z)] F(z)^{K-1} + [1 - F(z)^{K-1}] - \frac{K-1}{K} + \frac{K-1}{K} F(z)^K \\
&= F(z)^{K-1} - F(z)^K + 1 - F(z)^{K-1} - 1 + \frac{1}{K} + F(z)^K - \frac{1}{K} F(z)^K \\
&= \frac{1}{K} [1 - F(z)^K].
\end{aligned}$$

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